A LOW ORDER MODELLING TECHNIQUE FOR MULTIVARIABLE SYSTEMS

By

SANT SHARAN TEWARI

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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CERTIFICATE

This is to certify that the thesis entitled "A Low Order Modelling Technique for Multivariable Dynamic Systems" by Sant Sharan Tewari is a record of work carried out under my supervision and has not been submitted elsewhere for a degree.

Birendra Sahay

BIRENDRA SAHAY
Assistant Professor
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

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CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE A

Indexing Variables :

i, j, q - Integers

m - Degree of the denominator polynomial of a transfer function.

n - Degree of the numerator polynomial of a transfer function

p - number of input variables

s - number of output variables

k - An integer varying from 1 to s and identifying a particular output variable

l - An integer varying from 1 to p and identifying a particular input variable.

Variables :

A_{kq} - Data matrix for k^{th} output at q^{th} instant of time.

$a_{kl,o},\ldots,a_{kl,m}$ - Coefficients of the numerator polynomial of a transfer function relating kth output and lth input variables.

$b_{kl,1},\ldots,b_{kl,n}$ - Coefficients of the denominator polynomial of a transfer function relating kth output and lth input variables.

$c_{kl,i}=c_l[iT]$ - Change in the kth output variable due to lth input variable alone at time $t = iT$.

$c_{kl}(z)$ - z-transform of change in the kth output variable due to lth input variable alone.

$c_{k,i} = c_k[iT]$ - Total change in the kth output variable due to all the 'p' input variables acting together at time $t = iT$.

$C(z)$ - z-transform of total change in the kth output variable due to all the 'p' input variables acting together.

$\underline{C}(z)$ - A (sx1) vector containing 's' total output variables in their z-transforms as its elements.

$\underline{C}_{k,q}$ - A (qx1) vector containing kth total output variables at qth instant of time as its element.

$r_{l,i} = r_l[iT]$ - lth input variable at time $t = iT$.

$r_l(z)$ - z-transform of lth input variable.

$\underline{R}(z)$ - A (px1) vector containing 'p' input variables in their z-transform, as its elements.

T - Sampling interval.

$\underline{X}_k$ - A $[(m+n+1)P \times 1]$ vector containing the coefficients $a_{kl,o}, \ldots, a_{kl,m}$ and $b_{kl,1}, \ldots, b_{kl,n}$ for kth output and for $l = 1, \ldots, p$.

$\underline{X}_{kq}$ - A $[(m+n+1)P \times 1]$ vector containing the coefficients $a_{kl,o}, \ldots, a_{kl,m}$ and $b_{kl,1}, \ldots, b_{kl,n}$ for kth output and for $l = 1, \ldots, p$ at qth iteration.

$\hat{\underline{X}}_k$ - A $[(m+n+1)P \times 1]$ vector which best approximates the vector $\underline{X}_k$.

$\hat{\underline{X}}_{kq}$ - A $[(m+n+1)P \times 1]$ vector which best approximates t e vector $\underline{X}_{kq}$ at qth instant of time.

NOMENCLATURE B

A_1, A_2	=	cross-sectional area for main steam flow and secondary steam flow at the throttle valves (in^2.)
A_{K2}	=	constant which relates the flow and pressure drop in a turbine with many stages
C_1^*, C_2^*	=	coefficient used in determining steam flow through main and secondary throttle valves ($1b_m$/sec)
H_{FW}	=	heat transfer coefficient of feed water heaters (ws/lb)
H_R	=	heat transfer coefficient of reheater (ws/lb ^oF)
h_C, h_R, h_S, h_2', h_4'	=	enthalpy of main steam at nozzle chest, reheater, throttle valve, moisture separator and isentropic end points from pressure P_C and P_R, respectively (ws/lb)
h_f, h_{fw}', h_{fw}, h_0	=	enthalpy of saturated water in the reheater feed water leaving heater 1, heater 2 and water in the condenser respectively(ws/lb)
J, K_T	=	constant to relate mechanical energy and vice versa (ws/ftlb, ftlb/ws)
K_{BHP}, K_{BLP}	=	coefficient which indicates the amount of bled steam from the H.P. and L.P. turbines, respectively.

k_1, k_2	=	constant in the Callender's empirical equation relating pressure, density and enthalpy of superheated steam ($lb_f ft^3$/ $in.^2$ ws), lb_f $ft^3/in.^2$)
P, P_C, P_R	=	pressure in-reactor, nozzle chest and reheater ($lb/in.^2$)
Q_{H1}, Q_{H2}	=	heat transferred in heater no. 1 and 2, respectively (ws)
Q_R	=	heat transfer rate in the reheater (w)
R	=	gas constant (lb_f $ft^3/(in.^2$ lb_m ^oF))
T_{H1}, T_{H2}, T_{R1}, T_{R2}, T_{w2}, T_{w3}	=	time constants associated with heater no. 1, heater no. 2, reheaters for flow rate and heat transfer rate, and with the H.P. and L.P. turbines, respectively (1/sec)
T_{HP}, T_{LP}	=	torque of high pressure and low pressure turbines respectively (lb_f ft)
TORQUE	=	net torque at the turbine shafts (lb_f ft)
V_C, V_R	=	total effective volume of the nozzle chest and reheater, respectively (ft^3)
v_f, v_g	=	specific volume of saturated water and steam, respectively, around a pressure of 200 p.s.i. (cft/lb_m, cft/lb_m)

$w_1, w_2, w_3, w_2', w_2'$	=	steam flow rate at throttle valve, nozzle chest, L.P. turbine, before the reheater and after the reheater respectively (lb_m/sec)
w_{BHP}, w_{BLP}	=	bled steam from high pressure and low pressure turbines, respectively (lb_m/sec)
w_{FW}, w_{HP}, w_{MS}	=	water flow rate to reactor, collected in heater 2, and separated in the moisture separator (lb_m/sec)
w_L	=	a constant which depends on ratio of turbine flow required to produce a net torque to rated flow of steam
w_{PR}, w_{PR}^1	=	secondary steam flow rate entering and leaving the reheater, respectively (lb_m/sec)

Greek letters

Δ	=	denotes deviation from the operating point
η^*_{HP}, η^*_{LP}	=	isentropic efficiency of the H.P. and L.P. turbines, respectively
η_{CFHP}, η_{CFLP}	=	efficiency correction factors to take into account the reduction of rotational losses, etc. at reduced loads for H.P. and L.P. turbines, respectively

ρ_C, ρ_R, ρ_2 = density of steam in the nozzle chest, in the reheater and at the turbine outlet (lb/ft^3)

Ω = speed of turbine equal to 120 (rad/sec).

SYNOPSIS

A method is developed for low order modelling of large multivariable systems. The method is applicable to both linear as well as non-linear systems and does not require the transfer function or vector differential equation of the system to be specified. Only measured input-output data of the system at discrete intervals of time is required to determine its linear, low order, discrete-time model. The method is based on the use of matrix pseudo-inverse to estimate the parameters of the model which minimize the sum of the squares of the errors between the responses of the original system and the low order model at the sampling instants. A recursive algorithm is used which reduces the computational and data storage requirements and enables successively improved model of the system to be determined as new data from the system is made available. Thus the method can be used for on-line applications.

An iterative technique is used to convert the discrete-time model into a corresponding continuous-time model.

The method is used to obtain low order model of a nuclear reactor turbine system.

CHAPTER I

1.1 Introduction

Modern systems are often large, resulting in dynamic model which consists of a large number of non-linear differential equations. It is common practice to linearize the model around an operating point and assume a time-invariant behavior. The resulting linear model describes the behavior of the system adequately in the neighborhood of the operating point and in terms of computational effort the system simplifies considerably. In many situations, such as a thermal power plant, a nuclear reactor system, a process plant or aircraft systems etc., the dynamic model is still of such a large order as to be inconvenient or impractical for its simulation or control system design. Thus, it sometimes becomes important to look for a dynamically equivalent model which is of reduced order. The reduced order model should describe the behavior of the system almost as well as more complicated higher order model and retain the most important parameters of the original system.

In the last decade or so considerable attention has been devoted to the problem of deriving reduced-order models for complex systems. The problem is clearly of great interest to those working in the applications area [1], [2], [3].

1.2 Uses of reduced order models

Reduced-order models are obtained for the following reasons :

(1) To simplify understanding of the system, whether the problem is analysis, synthesis, or identification and to achieve easy simulation of the processes.

(2) To reduce computational requirements and to make better use of the supporting facilities as well as the computer itself, in addition to widening the scope of the problems which can be handled on a given size of machine. Of particular significance to on-line computer control applications is the reduction of the computational effort of optimal and adaptive controllers by deriving suboptimal strategies based on reduced order models [4], [5].

(3) To obtain a lower dimensional "control law" for simplifying the structure of the feedback controller [6].

(4) To make best use of scanty experimental data, by estimating a few parameter with confidence, rather than estimating more parameters with less confidence. The low order model is then a more reliable predictor [7].

(5) To generalize results established on a particular system to comparable systems, using low order models which basically differ in time scale only. This is

particularly useful in tolerancing performance for system check-out [8].

1.3 Literature survey

Although most of the work in the area of low order modelling of complex systems has been done in the last decade or so, the subject has aroused widespread interest. In the beginning different investigators presented radically new approaches to the same problem. Later on several variations and combinations of these approaches were developed. These different approaches along with their modifications are presented below grouped under different categories.

1.3.1 Modelling from loop data

The initial impetus came from Axelby [9] and Biernson [10] who developed methods for rapidly estimating system poles from the open loop Bode plot. This approach was formalised by Kan Chen [11], who suggested that dipoles might be estimated from system behavior in the high gain region, far-off poles from the low gain region, and dominant poles from the cross over region. Kan Chen method has been exploited by Canfield [12] for multiloop systems, and by Towill [13] in terms of a rational design procedure. The basic problem with these methods is that

if there are many break points in the high and low gain regions, the dominant poles, and hence the transient response of the low order model, is considerably in error.

1.3.2 Modelling from the system transfer function

Since the system transfer function contains all the data needed to model the system, there has always been effort to use the coefficients directly in a low order model. Deriving the system transfer function coefficients is time consuming if a computer sub-routine is not available, so that for high order systems it is not a preferred approach for hand computations. Oldenburger [14] neglected some of the coefficients on an intuitive partitioning basis and applied it to develop a reduced order model of a turbire governer system. Siljak [15], in parameter plane design methods, simply neglected the higher order coefficients and varied the lower order coefficients by adding a compensator to yield the original system response. The method due to Chen and Shieh [16] is based on continued fraction expansion (CFE) of the transfer function of the original system. The numerator and denominator polynomials in the transfer function are arranged in the ascending order, in powers of s, and then expanded into the Cauer-type continued fraction. From a consideration of the

final value theorem, it follows that quotients in the expansion are in the order of decreasing significance. Thus truncating the CFE at a suitable stage gives the reduced transfer function of the desired accuracy. The CFE used is equivalent to Taylor's series expansion about $s = 0$ and can be shown to be a special form of Pade' approximation for a scalor rational function [17]. The method ensures that the model gives the correct steady state response for a class of inputs, namely $\alpha_i \, t^i$, $i = 0, 1, 2, \ldots p$ where 'i' is a positive integer depending on the order of the model. However the initial transient response may not be good. Shamash [18], expanded the numerator and denominator into a Jordan-type continued fraction, about $s = 0$. This method requires much less computation than the original method. Chuang [19], [20] modified Chen and Shieh's method. He expanded the system transfer function into a Cauer-type continued fraction about $s = 0$ and $s = \infty$. By alternately expanding around $s = 0$ and $s = \infty$, this method ensures equally good steady state and initial transient responses. However, biased responses may also be obtained. By 'fitting' more coefficients in the power series expansion about $s = 0$ or $s = \infty$, a better steady state or initial transient response respectively, may be obtained.

Chen [21] extended his method to multivariable systems by expressing the transfer function matrix as a fraction with a matrix polynomial in the numerator and a scalor polynomial in the denominator in the ascending order and then expanding into matrix continued fraction as in the case of scalor systems. Truncating the continued fraction at a suitable stage gives the reduced order multivariable model. Chen applied this method to reduce the model of a gas-turbine system.

Vittal Rao and Lamba [22] minimised the integral of the weighted error function between a given transfer function and the reduced transfer function in the frequency domain. This method is of especial use in the non-linear systems where the frequency response of the linear part of the simplified system should approximate the frequency response of the linear part of the original system.

Shamash [23] extended the continued fraction methods for linear continuous-time systems to linear discrete-time systems.

1.3.3 Eigenvalue approach

Evans [24] is considered the originator of this approach as his root locus method brought the determination of eigenvalues into the realms of practicability for

realistic order systems. The first computerised, state-space approach to the problem is credited to Davison [25], and essentially consists of neglecting eigenvalues of the original system farthest from the origin, retaining only the dominant time constants. The retention of dominant eigenvalues makes the response of the reduced model approximate that of the original, since the eigenvalues neglected make a very insignificant contribution to the total response except at the beginning. Relationships from the time-solution of the vector differential equation of the original model were used to develop a reduced model which maintained both the correct proportion of the eigenvectors and the desired eigenvalues. The resulting low order model did not preserve steady state values, although the initial and middle parts of the responses were good representations. Marshall [26], by a similar reduction procedure and by neglecting the dynamics of fast modes of the original high order model, derived a lower order model which additionally achieved steady state comparability also and offered zero steady state error for constant inputs. Following the publication of Davison's work, there was much correspondence between Davison and Chidambara, but according to Marshall [27], no new method was proposed and the correspondence consisted essentially of variations upon the known theme.

Chidambara [28] accounted for the steady state error in Davison's method by pointing out the terms which cannot be neglected as steady state is approached. However, Davison [28] pointed out that Chidambara's method while giving correct steady state values, did not give good transient response agreement. This was because Chidambara was essentially finding a reduced forcing function and in doing so it did not excite modes of the approximate model in the same proportions as found in the original model. Chidambara [29], [30] countered Davison's arguments by giving examples. However, Davison [29], [30] pointed out that the examples given by Chidambara were misleading because only one mode was presented in the simplified model. He also mentioned that Chidambara's method was more difficult to use in practice. Following these publications, Davison [31] modified his method by defining a diagonal matrix and incorporating it in his original method in such a manner that the system gave satisfactory dynamic response and correct steady-state response for step inputs. Inspite of a number of these articles, it was not clear which model should be used since the examples given showed that either method is correct or incorrect according to various cases considered. Fossard [32] noted this and pointed out that the problem lied in the existence of null eigenvalues. He modified Davisons method to take

care of null eigenvalues and produced satisfactory steady state and dynamic responses for step input. He extended his method to general input case also by observing that if the slow modes of the system are correctly excited, those of the input will be incorrectly transmitted unless they are integrated into the system and the reduction process made on the augmented system. Newell and Fisher [33] applied Davison's method to reduce the order of an evaporator model. Seborg and Fisher [34] extended the continuous-time methods of Davison and Marshall to discrete time systems. An interesting variation to Davison's method is that proposed by Kuppurajulu and Elangovan [35] wherein the high order system is replaced by three models, successively representing the initial, intermediate and final stages of transient response by identifying those eigenvalues which are most dominant in the initial, intermediate and final portions of the system response. Mitra's [36] method consisted of orthogonal projection onto a subspace of the state space determined by a defined matrix. Wilson [37] determined a low order model by minimising a functional of error between the output vector of the system and the output vector of the model. The algorithm is based on the solution of a Lyapunov matrix equation and requires optimization of individual

parameters occurring in the particular canonical form chosen. However this method is cumbersome and quite time consuming. He extended it to multivariable systems also [38] and used it on a boiler model.

1.3.4 Modelling from response data

This is the general identification problem if we probe to find low order models which acceptably describe the 'black box' under test, through its input-output data. In the frequency domain, the historical path commenced with weighted least squares method of Levy [39]. He fitted the frequency dependent polynomial into the generated data by minimising an error function which was a measure of error in the fit. The method has been widely used and is capable of fitting some quite unusual frequency responses [40], but several difficulities have been noted with the method. Sanathanan and Koerner [41] found that the procedure described does not give a good fit if the frequency data spans several decades and have proposed instead an iterative procedure. The authors cited an example in which the iteratively-computed parameters differ considerably from those obtained by Levy's method. Sumner [42] has made the same criticism of the Levy method, but proposed a technique in which the error between the

data and the model is minimised using a non-linear optimisation method. A further modification made to the method by Payne [43] is to incorporate certain constraints based on additional knowledge of the system, such as steady state gain. He noted that in the unconstrained system there is a tendency for poles to occur on the right half of the complex plane and so give unstable responses in system known to be stable. He reported, in fact, that in fitting several hundred responses about 30% resulted in unstable systems when no constraints were used, but that the use of constraints reduced this to only 1 per cent. Towill and Payne [44] studied many systems and came to the conclusion that the least squares error criterion did not necessarily result in a satisfactory model in many systems. However he found this criterion to be most straightforward in terms of application and least computation time consuming in most of the cases. Considerable time may therefore be spent in system study and simulation until a satisfactory compromise is reached.

A number of methods minimising some function of the error between the system and the low order model at sampled response times have been developed. Anderson [45] determined a reduced order model, the response of which approaches that of the original in such a manner that the mean-square error between the two responses, over a given finite interval. is minimised. This is accomplished by

using the orthogonal projection theorem in the theory of linear vector spaces to minimize the sum of the squares of errors between the responses of the actual system and a discretized model at the sampling instants. The vector differential equation is then obtained from the vector difference equation of this discrete model. Sinha [46] derived a reduced order model by expressing the model of desired order in terms of a pulse transfer function, with its coefficients as unknowns. To find out these unknowns, he expressed the transfer function in an equivalent difference equation form. This equation when written for different time instances gave a set of ordinary algebraic equations with the coefficients of the reduced transfer function as unknowns. The equations were solved for these unknowns by calculating Left Minimum Pseudo-Inverse [47], [48] which was shown to give the unknown coefficients in such a manner that the sum of squares of the error between the original and the reduced model responses was minimized at the sampling instances. An iterative algorithm was used and thus the method required much less storage of data and much less computation than Anderson's method. Bereznai and Sinha [49] proposed another method based on the minimization of deviation between the

responses of the original system and the reduced order model in the direction that is everywhere perpendicular to the response of the system. He considered various error minimization criteria. However, he noted that analytical solution to the problem of error minimization is available only for the least squares error criterion. He suggested the pattern search techniques of Hooke and Jeeves [50] to solve problems formulated in other error minimization criteria. Meier and Luenberger [51] minimised the mean square difference between system output and model output when a random process is used to drive both systems. Hli [52] obtained a low order model from impulse response estimates by computing the time moments of the impulse response. The moments were then, using series expansion theory, equated to the unknown transfer function, and the coefficients determined by elimination. As the model order increases, considerable difficulty is experienced with convergence of the impulse moments for realistic test data. Nevertheless, it is a powerful technique for use in heavily damped systems. Riggs and Edgar [53] developed a general framework for optimum model reduction based on impulse responses of the original and reduced single input - single output systems and gave example of a heat diffusion system. Galiana [54] suggested a method of approximating linear time invariant

multi input - multi output systems by a lower order system through a weighted least squares minimization of a scalar integral function of the impulse response matrix over all positive time. A set of necessary conditions was derived in the form of a number of algebraic matrix equations, the solution of which yielded the unknown system parameters. Although he gave several examples, he did not apply it to any real system. Applevich's [55] method on multi-variable linear time-invariant discrete-time system was to optimally approximate the given system with respect to the sum of the squared output errors of the impulse response. He also established existence and uniqueness of the solution and in this connection developed certain projection theorems. Although he did not apply his method to a practical system, he gave an example.

1.3.5 Other approaches for low order modelling

Shieh and Wei [56] proposed a method which was a combination of dominant eigenvalue and continued fraction approaches. In their method which was applicable to multi-input multi-output systems, they expressed the transfer function matrix as a product of two polynomials. The dominant eigenvalues were used to formulate the common denominator polynomial of a reduced order model and the numerator dynamics of the reduced order model were

obtained by matrix-continued fraction approach. The method has the advantage of eigenvalue approach in the sense that reduced model is always stable. It also has the advantage that the computationally unattractive procedure of complicated linear transformation, matrix diagonalization, and steady state value matching, etc. involved in eigenvalue approach is replaced by a simple technique of continued fraction. Arumugam and Rama Moorty [57] used Schwarz canonical form in simplifying dynamics of systems described by linear differential equations. The method does not require the computation of eigenvalues and eigenvectors. But this method cannot be used for multivariable systems. Ravichandran [58] used singular perturbation method to reduce the model of a nuclear reactor system.

1.4 Present work

In the present work a method has been suggested to determine low order model of large multivariable system. This method is developed following the guidelines of Sinha's [46] work. Sinha developed a low order modelling technique from input-output data of a linear or non-linear system, measured at discrete intervals of time. Low order model is determined by using matrix pseudo-inverse technique which minimizes the squares of error between

the model and system responses at the sampling instants. However, this method was exclusively for single-input single-output systems. Moreover, this method was not applied to any real system. Sinha selected a fictitious linear system with its parameters chosen in such a manner as to simplify the analysis.

The present method is applicable to multivariable systems and can be used for on-line applications. The method is applicable to both linear and non-linear systems and does not require system transfer function or vector differential equation to be specified. Only the input-output data of the system measured at discrete intervals of time is required to determine the low order discrete time model. As in Sinha's method, matrix pseudo-inverse is used to minimize the squares of error between the responses of the low order model and the original system at sampling instants.

An iterative algorithm is used which reduces the computation and storage of data. The discrete time model is converted into corresponding continuous time model by application of Smith's [61] method.

The method is used to obtain low order model of a nuclear reactor turbine system.

Chapter II describes the mathematical development of the present method and the algorithm for obtaining the lower order model. In Chapter III, the model of the nuclear reactor turbine (which has been used here as an example) has been discussed and application of the present method has been developed. In Chapter IV, the computational results considering various orders of reductions have been given. Concluding remarks, limitations of the application of the method and scope for further work has been discussed in Chapter V.

CHAPTER II

PROBLEM FORMULATION AND METHOD OF SOLUTION

We aim at the determination of a low order model of a multivariable system from its input-output data obtained at discrete intervals of time. Such a case might arise in modelling of a large system by a low order model, which has considerably smaller order dynamics, by using directly the measured input-output data.

Once the algorithm to find lower order model from actual input-output data has been incorporated in the computer, it will give successively improved lower order model as density of the input-output from the plant increases. This can be used as an on-line process.

2.1 Problem formulation

The low order model of a system may be represented by a block diagram shown in Fig. 2.1. For the 'p' inputs $r_1(s)$, $r_2(s)$, ... $r_p(s)$, there may be 'S' outputs $C_1(s)$, $C_2(s)$, ... $C_s(s)$ of the model. The relationship between the input and output variables is described by transfer function matrix $H(s)$, which in effect describes the low order model.

If the system is modelled as low order <u>linear</u> model, each input can be treated independently of each other. Complete output of the system can then be obtained

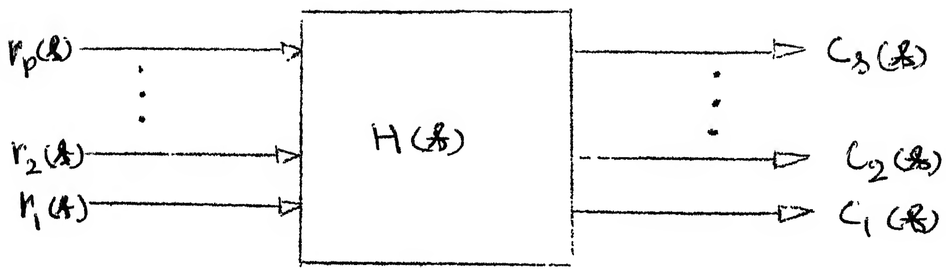

Figure 2.1 Multi-Input Multi-Output System

by superposition, i.e., outputs corresponding to each input alone are added together. Thus, the i-th output $C_i(s)$ is given by,

$$C_i(s) = \sum_{j=1}^{p} H_{ij}(s) r_j(s) \; ; \; i = 1,2,\ldots s \qquad (2.1)$$

where $r_j(s)$ is the j-th input and $H_{ij}(s)$ is the transfer function between i-th output and j-th input with all other inputs reduced to zero. For different values of i and j, this describes the transfer function matrix with each $H_{ij}(s)$ as an element transfer function. Now, the modelling process reduces to finding coefficients of each of the transfer function such that the output variables of the low order model approximate those of the original system.

2.1.1 Low order modelling for discrete system data

Since in practice it is easier to make measurements at discrete intervals of time than to measure them continuously, the system is modelled as low order discrete model. The model will be derived so that its output approximate those of the original system at the sampling instants. Such a model can be described by a digital element wherein the output is in the form of pulse sequences.

In a digital element, the difference equation relating the input and output pulse sequences is,

$$c_{kl}\left[iT\right] - b_{kl,1}\, c_{kl}\left[(i-1)T\right] - b_{kl,2}\, c_{kl}\left[(i-2)T\right] - \dots - b_{kl,n}\, c_{kl}\left[(i-n)T\right]$$

$$= a_{kl,o}\, r_l\left[iT\right] + a_{kl,1}\, r_l\left[(i-1)T\right] + a_{kl,2}\, r_l\left[(i-2)T\right] + \dots + a_{kl,m}\, r_l\left[(i-m)T\right] \qquad (2.2)$$

where,

r_l is the l^{th} input variable.

c_{kl} is the change in the k^{th} output variable due to l^{th} input alone.

$b_{kl,1}, \dots, b_{kl,n}$ and $a_{kl,o}, \dots a_{kl,m}$ etc. are the various coefficients.

T is the sampling interval

i is an integer.

This linear relation can be interpreted as a formula through which the present output number $c_{kl}(iT)$ can be computed by taking weighted sums of a fixed group of input and output numbers. The process of digital element is more clearly depicted in the Fig. 2.2.

The output c_{kl} at any time $t = iT$ depends on input r_l applied at $t = iT$ as well as on the previous outputs at times $(i-1)T$, $(i-2)T, \dots (i-n)T$ and on the previous

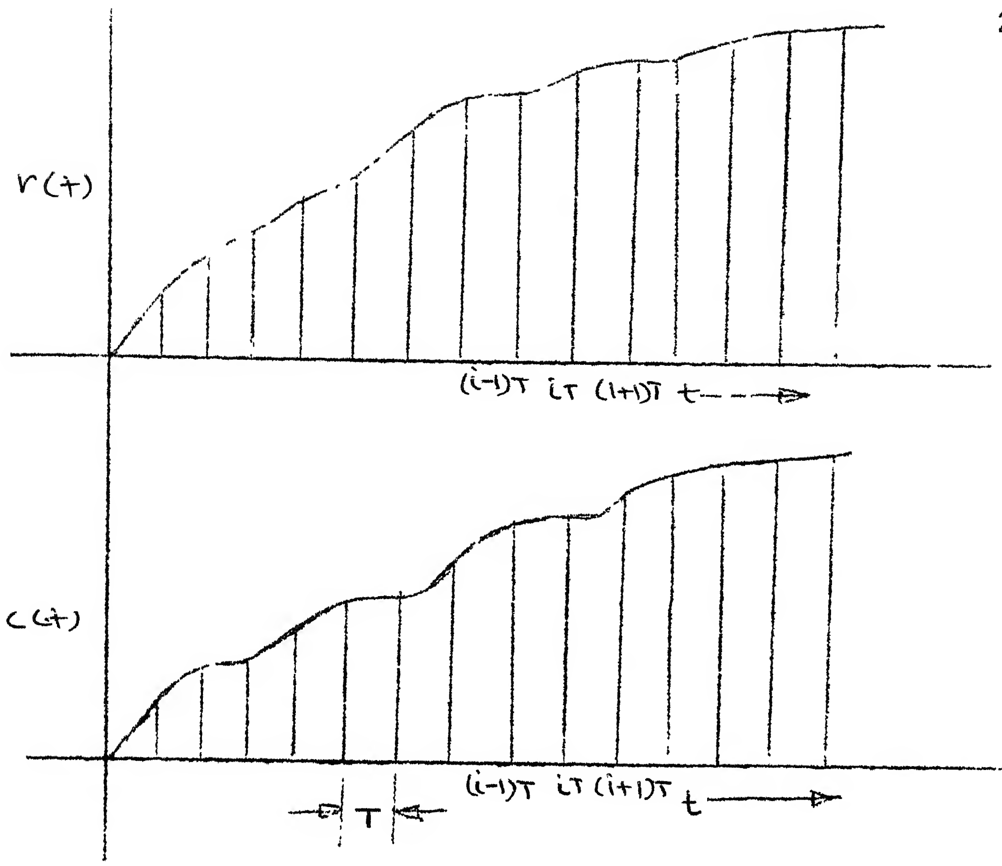

Figure 2.2 Input-Output Pulse Sequences In A Digital Element

inputs applied at times $(i-1)T$, $(i-2)T, \ldots, (i-m)T$. The relative effects of these variables in the previous time values on the present output variable $c_{kl}(iT)$ is given by the weighting coefficients $b_{kl,1} \ldots b_{kl,n}$ and $a_{kl,o} \ldots a_{kl,m}$.

In a linear multi-input multi-output system, superposition theorem can be applied; so that the outputs corresponding to each input alone are added together to give the total output. In other words,

$$C_k\,(iT) = \sum_{l=1}^{p} c_{kl}\,(iT)\ ;\ k = 1,2,\ldots,s \tag{2.3}$$

where,

$C_k\,(iT)$ is the k-th total output at $t = iT$ when 'p' inputs are applied.

$c_{kl}\,(iT)$ is the k-th output at $t = iT$ corresponding to l-th input alone.

So, in the multivariable, linear discrete system, the appropriate difference equation which relates the input and output pulse sequences is obtained by taking summation on both sides of eqn. (2.2) from 1 to p, and is,

$$C_k\,[iT] = \sum_{l=1}^{p} c_{kl}\,[iT] = \sum_{l=1}^{p} (a_{kl,o}\, r_l[iT] + a_{kl,1}\, r_l[(i-1)T] + \ldots + a_{kl,m}\, r_l[(i-m)T]) + \sum_{l=1}^{p} (b_{kl,1}\, c_{kl}[(i-1)T] + \ldots + b_{kl,n}\, c_{kl}[(i-n)T])\ ;\ k = 1,2,\ldots,s \tag{2.4}$$

2.1.2 Z-transform analysis of the system

In order to describe the dynamics of the discrete process represented by the linear difference equations given in the previous section, Z-transform analysis is used.

Multiplying both sides of eqn. (2.2) by e^{-iTs}, where s is the Laplace-transform variable, and taking the summation from i = 0 to ∞, we have,

$$\sum_{i=0}^{\infty} c_{kl}[iT]\, e^{-iTs} - \sum_{i=0}^{\infty} b_{kl,1}\, c_{kl}[(i-1)T]\, e^{-iTs}$$
$$- \sum_{i=0}^{\infty} b_{kl,2}\, c_{kl}[(i-2)T] e^{-iTs} - \dots -$$
$$\sum_{i=0}^{\infty} b_{kl,n}\, c_{kl}[(i-n)\, T] e^{-iTs} = \sum_{i=0}^{\infty} a_{kl,o}\, r_l[iT] e^{-iTs} +$$
$$\sum_{i=0}^{\infty} a_{kl,1}\, r_l[(i-1)T]\, e^{-iTs} + \sum_{i=0}^{\infty} a_{kl,2}\, r_l[(i-2)T] e^{-iTs}$$
$$+ \dots + \sum_{i=0}^{\infty} a_{kl,m}\, r_l[(i-m)T] e^{-iTs} \qquad (2.5)$$

The lower index in the summation of (2.5) is zero, which indicates the fact that the values of the sequences for the negative time are zero.

Putting in eqn. (2.5), $Z = e^{Ts}$ from the definition of Z-transform and making use of the 'Shifting theorem' which states,

$$Z^{-j}\, r_1[iT] = r_1[(i-j)T]$$

and $$Z^{-j}\, c_{kl}[iT] = c_{kl}[(i-j)T].$$

$$\sum_{i=0}^{\infty} c_{kl}[iT] Z^{-i} - b_{kl,1} \sum_{i=0}^{\infty} c_{kl}[iT]\, Z^{-i-1} - b_{kl,2} \sum_{i=0}^{\infty} c_{kl}[iT] Z^{-i-2}$$

$$- \cdots - b_{kl,n} \sum_{i=0}^{\infty} c_{kl}[iT] Z^{-i-n} = a_{kl,o} \sum_{i=0}^{\infty} r_1[iT] Z^{-i}$$

$$+ a_{kl,1} \sum_{i=0}^{\infty} r_1[iT]\, Z^{-i-1} + \cdots + a_{kl,m} \sum_{i=0}^{\infty} r_1[iT] Z^{-i-m} \qquad (2.6)$$

Factoring out the common summation for each of the terms of eqn. (2.6), these results,

$$\sum_{i=0}^{\infty} c_{kl}[iT] Z^{-i} (1 - b_{kl,1}\, Z^{-1} - b_{kl,2}\, Z^{-2} - \cdots - b_{kl,n}\, Z^{-n}) =$$

$$\sum_{i=0}^{\infty} r_1[iT] Z^{-i} (a_{kl,o} + a_{kl,1}\, Z^{-1} + \cdots + a_{kl,m}\, Z^{-m}) \qquad (2.7)$$

Again from the definition of Z-transform,

$$c_{kl}(Z) = \sum_{i=0}^{\infty} c_{kl}[iT]\, Z^{-i}$$

and $$r_1(Z) = \sum_{i=0}^{\infty} r_1[iT] Z^{-i}$$

Using these definitions in eqn. (2.7), we have,

$$c_{kl}(Z)\ (1-b_{kl,1}\ Z^{-1} - b_{kl,2}\ Z^{-2} - \ldots - b_{kl,n}\ Z^{-n}) =$$

$$r_l\ (Z)\ (a_{kl,o} + a_{kl,1}\ Z^{-1} + \ldots + a_{kl,m}\ Z^{-m}) \qquad (2.8)$$

or,

$$\frac{c_{kl}\ (Z)}{r_l\ (Z)} = H_{kl}(Z) = \frac{a_{kl,o} + a_{kl,1}\ Z^{-1} + \ldots + a_{kl,m}\ Z^{-m}}{1-b_{kl,1}\ Z^{-1} + \ldots + a_{kl,n}\ Z^{-n}} \qquad (2.9)$$

This expression gives the transfer function between the k-th output and the l-th input. For different values of k and l this describes the transfer function matrix $H_{kl}(Z)$ with each transfer function $H_{kl}\ (Z)$ as its element.

To express eqn. (2.9) in the transfer function matrix form, we write it separately for each input variable i.e. for each value of index 'l', for a given output. The total output variable is obtained by using superposition theorem.

$$C_k\ (Z) = \sum_{l=1}^{p} c_{kl}\ (Z) = \sum_{l=1}^{p} H_{kl}\ (Z)\ r_l\ (Z) \qquad (2.10)$$

Writing eqn. (2.10) for different values of 'k' transfer function matrix $H_{kl}\ (Z)$ is obtained.

2.1.3 A particular case

An example of a multivariable system having two input output variables follows :

Here $l = 1,2$

$$\text{So, } c_{k1}(Z) = \frac{a_{k1,o} + a_{k1,1}Z^{-1} + \ldots + a_{k1,m}Z^{-m}}{1 - b_{k1,1}Z^{-1} - b_{k1,2}Z^{-2} - \ldots - b_{k1,n}Z^{-n}}\, r_1(Z); k=1,2$$

$$\text{and } c_{k2}(Z) = \frac{a_{k2,o} + a_{k2,1}Z^{-1} + \ldots + a_{k2,m}Z^{-m}}{1 - b_{k2,1}Z^{-1} - b_{k2,2}Z^{-2} - \ldots - b_{k2,n}Z^{-n}}\, r_2(Z); k=1,2$$

These are the output variables due to inputs $r_1(Z)$ and $r_2(Z)$ respectively. The total output variable is,

$$C_k(Z) = C_{k1}(Z) + c_{k2}(Z) \; ; \; k = 1,2$$

Thus the system is represented in the following transfer function matrix form -

$$\begin{bmatrix} c_{11}(Z)+c_{12}(Z) \\ c_{21}(Z)+c_{22}(Z) \end{bmatrix} = \begin{bmatrix} C_1(Z) \\ C_2(Z) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{a_{11,o}+a_{11,1}Z^{-1}+\ldots+a_{11,m}Z^{-m}}{1-b_{11,1}Z^{-1}-\ldots-b_{11,n}Z^{-n}} & \dfrac{a_{12,o}+a_{12,1}Z^{-1}+\ldots+a_{12,m}Z^{-m}}{a-b_{12,1}Z^{-1}-\ldots-b_{12,n}Z^{-n}} \\ \dfrac{a_{21,o}+a_{21,1}Z^{-1}+\ldots+b_{21,m}Z^{-m}}{1-b_{21,1}Z^{-1}-\ldots-b_{21,n}Z^{-n}} & \dfrac{a_{22,o}+a_{22,1}Z^{-1}+\ldots+a_{22,m}Z^{-m}}{1-b_{22,1}Z^{-1}-\ldots-b_{22,n}Z^{-n}} \end{bmatrix} \begin{bmatrix} r_1(Z) \\ r_2(Z) \end{bmatrix}$$

$$\text{or} \qquad \underline{C}(Z) = H(Z)\;\underline{R}(Z)$$

2.1.4 General transfer function matrix

As in the section (2.1.3), the eqn. (2.9) may be used to write the transfer function matrix of the model having 'p' inputs and 's' outputs. So, the reduced order, linear discrete system may be expressed in the pulse transfer function matrix form :

$$\underline{C}\,(Z) = H(Z)\ \underline{R}(Z), \tag{2.11}$$

where,

$$\underline{C}^T(Z) = [C_1\ \ C_2\ \cdots\ C_s], \tag{2.12}$$

$$\underline{R}^T(Z) = [r_1\ \ r_2\ \cdots\ r_p], \tag{2.13}$$

$$H(Z) = \begin{bmatrix} \dfrac{a_{11,0}+\cdots+a_{11,m}z^{-m}}{1-b_{11,1}z^{-1}-\cdots-b_{11,n}z^{-n}} & \dfrac{a_{12,0}+\cdots+a_{12,m}z^{-m}}{1-b_{12,1}z^{-1}-\cdots-b_{12,n}z^{-n}} & \cdots & \dfrac{a_{1p,0}+\cdots+a_{1p,m}z^{-m}}{1-b_{1p,1}z^{-1}-\cdots-b_{1p,n}z^{-n}} \\ \dfrac{a_{21,0}+\cdots+a_{12,m}z^{-m}}{1-b_{21,1}z^{-1}-\cdots-b_{21,n}z^{-n}} & \dfrac{a_{22,0}+\cdots+a_{22,m}z^{-m}}{1-b_{22,1}z^{-1}-\cdots-b_{22,n}z^{-n}} & \cdots & \dfrac{a_{2p,0}+\cdots+a_{2p,m}z^{-m}}{1-b_{2p,1}z^{-1}-\cdots-b_{2p,n}z^{-n}} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{a_{s1,0}+\cdots+a_{s1,m}z^{-m}}{1-b_{s1,1}z^{-1}-\cdots-b_{s1,n}z^{-n}} & \dfrac{a_{s2,0}+\cdots+a_{s2,m}z^{-m}}{1-b_{s2,1}z^{-1}-\cdots-b_{s2,n}z^{-n}} & \cdots & \dfrac{a_{sp,0}+\cdots+a_{sp,m}z^{-m}}{1-b_{sp,1}z^{-1}-\cdots-b_{sp,n}z^{-n}} \end{bmatrix} \tag{2.14}$$

of from eqn. (2.4), in the form of the equivalent difference equation :

$$C_{k,i} = \sum_{l=1}^{p} \sum_{j=0}^{m} a_{kl,j}\, r_{l,i-j} + \sum_{l=1}^{p} \sum_{j=1}^{n} b_{kl,j}\, c_{kl,i-j}\,; \qquad k = 1,2,\ldots s \qquad (2.15)$$

$$= \sum_{j=0}^{m} \sum_{l=1}^{p} a_{kl,j}\, r_{l,i-j} + \sum_{j=1}^{n} \sum_{l=1}^{p} b_{kl,j}\, c_{kl,i-j}\,; \qquad k = 1,2,\ldots,s \qquad (2.16)$$

2.1.5 Matrix equation representation

Letting i range from zero to some integer 'q', eqn. (2.16) may be represented by the matrix equation :

$$A_{kq}\, \underline{X}_k = \underline{C}_{k,q} \qquad (2.17)$$

where :

$$\underline{X}_k^T = \left[(a_{k1,0}\ a_{k2,0} \ldots a_{kp,0})\ (a_{k1,1}\ a_{k2,1} \ldots a_{kp,1}) \ldots (a_{k1,m}\ a_{k2,m} \ldots a_{kp,m})\ (b_{k1,1} \ldots b_{kp,1})\ (b_{k1,2}\ b_{k2,2} \ldots b_{kp,2}) \ldots (b_{k1,n}\ b_{k2,n}, \ldots b_{kp,n}) \right] \qquad (2.18)$$

$$\underline{C}_{k,q}^T = \left[C_{k,1}\ C_{k,2} \ldots C_{k,q} \right] \qquad (2.19)$$

$$A_{kq} = \begin{bmatrix} (r_{1,0} \cdots r_{p,0}) \cdots (r_{1,-m} \cdots r_{p,-m}) & (c_{k1,-1} \cdots c_{kp,-1}) \cdots (c_{k1,-n} \cdots c_{kp,-n}) \\ (r_{1,1} \cdots r_{p,1}) \cdots (r_{1,1-m} \cdots r_{p,1-m}) & (c_{k1,0} \cdots c_{kp,0}) \cdots (c_{k1,1-n} \cdots c_{kp,1-n}) \\ \vdots & \vdots \\ (r_{1,q} \cdots r_{p,q}) \cdots (r_{1,q-m} \cdots r_{p,q-m}) & (c_{k1,q-1} \cdots c_{kp,q-1}) \cdots (c_{k1,q-n} \cdots c_{kp,q-n}) \end{bmatrix} \tag{2.20}$$

Vector $\underline{X}_k$ is termed 'Parameter' vector and matrix A_{kq} is termed 'Data' matrix.

The problem, then, is the determination of $(m + n + 1)p$ components of the Parameter vector $\underline{X}_k$ from the matrix eqn. (2.17) the left hand side of which contains the Data matrix, obtained from the input-output data at sampling instants. Its right hand side is the vector of total output variables, all of which are known. Therefore, the problem simply appears to be that of the solution of a set of linear algebraic equations. However, in practice, the number of rows in the Data matrix is much larger than the number of columns. This is particularly so when the data is taken over a large interval and the sampling rate is high. Moreover since we want the order of the reduced model to be as small as possible, the number of columns in the Data matrix will naturally be small. Because of this reason we get more number of equations than are the number of unknowns, and the set of linear equations represented by (2.17) does not possess a unique solution, unless m and n are the actual orders of the polynomials of the pulse transfer functions. As this is not the case, by hypothesis, we can determine that parameter vector, $\hat{\underline{X}}_k$, which gives the 'best' fit. One suitable criterion for an optimum fit is the minimization of the sum of the

squares of the errors between the responses of the actual system and the reduced model at the sampling instants. This is the criterion considered in the present work.

2.2 Method of solution

As stated earlier, the problem does not have a unique solution. In other words, since the Data matrix is rectangular, it does not have an inverse in the ordinary sense. However, for $j > (m + n + 1)\,p$, the least-squares solution to eqn. (2.17) is given by,

$$\hat{\underline{X}}_k = A_{kq}^{LM} \; \underline{C}_q \qquad (2.21)$$

where A_{kq}^{LM} is called 'Left Minimum Pseudo Inverse' of the matrix A_{kq} and is defined by,

$$A_{kq}^{LM} = \left[(A_{kq}^{LM})^T \; A_{kq} \right]^{-1} \cdot A_{kq}^T \qquad (2.22)$$

The properties of the matrix pseudo-inverse have been discussed thoroughly by Penrose [47] and Greville [48]. A description of matrix pseudo-inverse is also given in the appendix. There it has been shown that $\hat{\underline{X}}_k$, from eqn. (2.21) is determined in such a manner that the sum of the squares of error between the right hand side of eqn. (2.17) (which is the measured response of the actual system) and its left hand side (which is response of the low order model) is minimized. In other words, pulse transfer

functions in the transfer function matrix (2.14), with the coefficients determined from (2.18), yield responses of the low order model which approximate those of the original system.

2.3 Algorithms based on the matrix pseudo-inverse

In practice, the Data matrix is quite large and it is not feasible to calculate A_{kq}^{LM} directly. This is particularly the case when a large amount of data must be stored in order to minimize the mean-square error over a sufficiently large interval, especially when the sampling rate is high. This difficulty can be overcome by using the following recursive algorithm, originally proposed by Albert and Sittler [59] and modified by Wells [60] for the case where a row is added to A_{kq} every time, as happens with each pair of additional input output data:

Let :

$$A_{kq+1} = \begin{bmatrix} A_{kq} \\ \underline{a}^T_{kq+1} \end{bmatrix} \qquad (2.23)$$

where

$$\underline{a}^T_{kq+1} = \Big[(n_{1,q+1}\ r_{2,q+1} \cdots r_{p,q+1})(r_{1,q}\ r_{2q} \cdots r_{p,q}) \cdots (r_{1,q-m+1} r_{2,q-m+1}, \cdots r_{p,q-m+1})(c_{k1,q}\ c_{k2,q} \cdots c_{kp,q-p+1}) (c_{k1,q-1}\ c_{k2,q-1} \cdots c_{kp,q-1}) \cdots (c_{k1,q-n+1} c_{k2,q-n+1} \cdots c_{kp,q-n+1})\Big] \qquad (2.24)$$

and let :

$$\underline{C}_{k,q+1} = \begin{bmatrix} \underline{C}_{k,q} \\ C_{k,q+1} \end{bmatrix} \tag{2.25}$$

The recursive algorithm that follows is divided in two phases. The first phase is the starting phase. We start from q = 0 and go upto $q \leq (m + n + 1)\, p$ and use the following recursive relations :

$$\hat{\underline{X}}_{k,q+1} = \hat{\underline{X}}_{kq} + \frac{Q_{kq}\, \underline{a}_{k,q+1} (C_{k,q+1} - \underline{a}^T_{k,q+1}\, \hat{\underline{X}}_{k,q})}{\underline{a}^T_{k,q+1}\, Q_{k,q}\, \underline{a}_{k,q+1}} \tag{2.26}$$

where

$$Q_{k,q+1} = Q_{k,q} - \frac{Q_{k,q}\, \underline{a}_{k,q+1} \left[Q_{k,q}\, \underline{a}_{k,q+1}\right]^T}{\underline{a}^T_{k,q+1}\, Q_{k,q}\, \underline{a}_{k,q+1}} \tag{2.27}$$

and

$$P_{k,q+1} = P_{k,q} - \frac{\left[P_{k,q}\, \underline{a}_{k,q+1}\right]\left[P_{k,q}\, \underline{a}_{k,q+1}\right]^T + \left[Q_{k,q}\underline{a}_{k,q+1}\right]\left[P_{k,q}\underline{a}_{k,q+1}\right]^T}{\underline{a}^T_{k,q+1}\, Q_{k,q}\, \underline{a}_{k,q+1}}$$

$$+ \frac{\left[Q_{k,q}\underline{a}_{k,q+1}\right]\left[Q_{k,q}\, \underline{a}_{k,q+1}\right]^T \left[1 + \underline{a}^T_{k,q+1}\, P_{k,q}\, \underline{a}_{k,q+1}\right]}{(\underline{a}^T_{k,q+1}\, Q_{k,q}\, \underline{a}_{k,q+1})^2} \tag{2.28}$$

with the initial conditions $\hat{\underline{X}}_o = \underline{0}$, $Q_o = I$ and $P_o = 0$.

The second phase follows the first and starts for $q \geq (m+n+1)p$. The recursive relations in this phase are :

$$\hat{\underline{X}}_{k\,q+1} = \hat{\underline{X}}_{k,q} + \frac{P_{k,q}\, \underline{a}_{k,q+1}\, (C_{k,q+1} - \underline{a}^T_{k,q+1}\, \hat{\underline{X}}_{kq})}{1 + \underline{a}^T_{k,q+1}\, P_{k,q}\, \underline{a}_{k,q+1}} \tag{2.29}$$

where

$$P_{k,q+1} = P_{k,q} - \frac{\left[P_{k,q}\, \underline{a}_{k,q+1}\right]\left[P_{k,q}\, \underline{a}_{k,q+1}\right]^T}{1 + \underline{a}^T_{k,q+1}\, P_{k,q}\, \underline{a}_{k,q+1}} \tag{2.30}$$

2.4 Determination of the equivalent continuous-time model

The algorithm given previously determine the low-order pulse transfer function matrix H(z) for the system. The problem now is to determine the corresponding continuous-time system transfer function matrix H(s). It should be noted that H(s) is not the Laplace transform of the impulse response which has the z-transform H(z). The relationship between H(s) and H(z) depends upon the form of the input between the sampling instants. Anderson[45] has assumed that the input is held constant between the sampling instants. This is equivalent to the introduction of a zero-order hold, so that H(z) may be regarded as the z-transfer function of H(s) preceded by a sampler and a zero-order hold. If the input to the system is a unit step this does not introduce any errors at all.

Smith [61] has given a method for estimating the Laplace-transform coefficients of a dynamic system from sampled-data values of its input and output. Advantage of this method is that it is quite straight

forward and can be easily programmed in a computer as the algorithm is iterative in nature. The method is applicable to the systems with noise. Its application on noise-free systems is described in section 2.4.1.

2.4.1 Description of Smith's method

In this method the dynamic system is assumed to be linear with a Laplace transform of the form,

$$H(s) = \frac{\alpha_0 + \alpha_1 s + \dots + \alpha_p s^p}{\beta_0 + \beta_1 s + \dots + \beta_q s^q} \tag{2.47}$$

The coefficients $\alpha_0, \alpha_1, \dots, \alpha_p$ and $\beta_0, \dots, \beta_q$ in this transfer function are not known. The pulse transfer function $\hat{H}(z)$ which exactly relates the input and output sampled data sequences is

$$\hat{H}(z) = \frac{\hat{a}_0 + \hat{a}_1 z^{-1} + \dots + \hat{a}_m z^{-m}}{\hat{b}_0 + \hat{b}_1 z^{-1} + \dots + \hat{b}_n z^{-n}} \tag{2.48}$$

In general each of the coefficient in this pulse transfer function is a function of coefficients of the transfer function (3.47). In other words,

$$\hat{a}_o = \hat{a}_o\,(\alpha_o, \alpha_1, \ldots, \alpha_p\ \ \beta_o,\ \beta_1, \ldots,\ \beta_q)$$
$$\vdots$$
$$\hat{a}_m = \hat{a}_m\,(\alpha_o, \alpha_1, \ldots \alpha_p,\ \beta_o,\ \beta_1, \ldots,\ \beta_q)$$
$$\hat{b}_o = \hat{b}_o\,(\alpha_o, \alpha_1, \ldots, \alpha_p,\ \beta_o,\ \beta_1, \ldots,\ \beta_q) \tag{2.49}$$
$$\vdots$$
$$\hat{b}_n = \hat{b}_n\,(\alpha_o, \alpha_1, \ldots, \alpha_p,\ \beta_o,\ \beta_1, \ldots,\ \beta_q)$$

Now, the pulse transfer function (2.48) must be of the same form as the pulse transfer function H(z), estimated from the data. If it is not, then the form of H(s) is changed in such a manner that the forms of the pulse transfer functions $\hat{H}(z)$ and H(z) are exactly similar.

The pulse transfer function H(z) estimated from data is given by,

$$H(z) = \frac{a_o + q_1 z^{-1} + \ldots + a_m z^{-m}}{b_o + b_1 z^{-1} + \ldots + b_n z^{-n}} \tag{2.50}$$

The coefficients, a_o, $a_1, \ldots,$ a_m and b_o, $b_1, \ldots,$ b_n of this transfer function are known.

Now equating the coefficients of H(z) and H(z) for corresponding powers of Z, we get a system of equations, with $\alpha_o, \alpha_1, \ldots, \alpha_p$ and β_o, $\beta_1, \ldots,$ β_q as

unknowns,

$$\hat{a}_o(\alpha_o, \alpha_1, \ldots, \alpha_p, \beta_o, \beta_1, \ldots, \beta_q) = a_o$$

$$\vdots$$

$$\hat{a}_m(\alpha_o, \alpha_1, \ldots, \alpha_p, \beta_o, \beta_1, \ldots, \beta_q) = a_m$$

$$\hat{b}_o(\alpha_o, \alpha_1, \ldots, \alpha_p, \beta_o, \beta_1, \ldots, \beta_q) = b_o \tag{2.51}$$

$$\vdots$$

$$\hat{b}_n(\alpha_o, \alpha_1, \ldots, \alpha_p, \beta_o, \beta_1, \ldots, \beta_q) = b_n$$

Solving eqns. (2.51) for the unknowns, $\alpha_o, \alpha_1, \ldots, \alpha_p$ and $\beta_o, \beta_1, \ldots, \beta_q$, the coefficients of the transfer function H(s) are obtained. An example of this method is given in section 2.4.2.

2.4.2 An example of the application of Smith's method

Suppose that the pulse transfer function obtained from sampled data is,

$$H(z) = \frac{b_o + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{2.52}$$

Here, the values of coefficients b_o, b_1, b_2, a_1 and a_2 are known. The form of this transfer function suggests that H(s) to be estimated from it should have the form [64],

$$H(s) = \frac{\alpha_o + \alpha_1 s}{s^2 + \beta_1 s + \beta_2} \tag{2.53}$$

where,

$$b_o = \frac{\alpha_o}{\beta_2} \tag{2.54}$$

$$b_1 = \frac{\alpha_o}{\beta_2} e^{-\frac{\beta_1 T}{2}} \Big[-2 \cos (.5\sqrt{4\beta_2^2 - \beta_1^2}\, T + \emptyset)$$

$$+ A \sin (.5\sqrt{4\beta_2^2 - \beta_1^2}\, T + \emptyset) \Big] \tag{2.55}$$

$$b_2 = \frac{\alpha_o}{\beta_2} \; e^{-\frac{\beta_0 T}{2}} \Big[e^{-\frac{\beta_1 T}{2}} - A \sin (.5\sqrt{4\beta_2^2 - \beta_1^2}\, T + \emptyset) \Big] \tag{2.56}$$

$$a_1 = -2 \, e^{-\frac{\beta_1 T}{2}} \cos(.5\sqrt{4\beta_2^2 - \beta_1^2}\, T + \emptyset) \tag{2.57}$$

$$a_2 = e^{-\beta_1 T} \tag{2.58}$$

here, $A = \frac{1}{\sqrt{1 - \beta_1^2 / 4\beta_2}} \sqrt{1 \quad \frac{\alpha_o}{\alpha_1} \beta_1 + \frac{\alpha_o^2}{\alpha_1^2} \beta_2}$

and $\emptyset = \tan^{-1} \left(\frac{.5\, \alpha_o \sqrt{4\beta_2^2 - \beta_1^2}}{\alpha_1 - .5\alpha_o \beta_1} \right) - \tan^{-1} \left(\frac{\sqrt{1 - \frac{\beta_1^2}{4\beta_2}}}{-\beta_1 / 2\sqrt{\beta_2}} \right)$

As can be seen, there are four unknowns, α_o, α_1, β_1 and β_2 and five equations relating these variables are available. Thus exact values of all of these coefficients cannot be determined. To get approximate

values, of following iterative scheme may be used,

Step I - Find β_1 from eqn. (2.58)

Step II - Assume a value of β_2 and find α_0 from eqn. (2.54).

Step III- Combine eqns. (2.56) and 2.57) to eliminate trignometrical expressions and by substituting the values of β_1, β_2 and α_0, find α_1.

Step IV - Substitute values of α_1, β_1, β_2 in eqn.(2.55) and get a value of α_0.

Step V - Compare the values of α_0 obtained in Step II and step IV. If they are close enough, stop, otherwise, assume a new value of β_2 and repeat the above steps.

A computer program can be made to implement the above scheme.

CHAPTER III

APPLICATION TO A NUCLEAR REACTOR TURBINE MODEL

3.1 Brief description of the nuclear reactor turbine model

The development of the model of the nuclear reactor turbine given here is based on IBM report [62] and reference [63].

The block diagram representation of the turbine coupled to the nuclear reactor with its various elements is shown in Fig. 3.1. The steam turbine assembly can be divided into various subsystems like high pressure, intermediate pressure, low pressure units, feed water heaters, etc. The well known process representation for this part of the modified Rankine cycle is represented in Fig. 3.2. The steam flowing from the reactor through the main steam valve is just dry, hence the steam exiting from the high pressure turbine will be wet, as indicated by the condition of steam at point 2 of Fig. 3.2. The water part of the wet steam is removed in the moisture separator and the dry steam at pressure p_R is superheated to temperature T_R in the reheater by using secondary steam drawn from the main supply line through a Secondary Steam Valve. This superheated steam expands through intermediate pressure and low pressure turbines and then enters the condenser. The condensed steam, i.e. feed water,

flows through the feed water heaters before it enters the steam drum. The feed pump in between the two heaters raises the pressure from the condenser pressure to the reactor drum pressure. For the above system the model and plant dynamics are described below.

3.2 Model and the Plant dynamics

The model was developed by the application of well known physical laws and a few pertinent empirical relationships. Various assumptions made in the development of the model are presented first, followed by the model itself.

(1) The relationship between enthalpy, specific volume and pressure (i.e. the equation of state) for the superheated steam follows Callender's empirical relationship for all non-constant pressure processes.

(2) The I.P. and L.P. turbine dynamics are lumped together, as both have a correspondingly slower steam flow dynamics as compared to the H.P. turbine.

(3) The flow through the throttle valve is assumed to depend on the square root of the upstream pressure and density of the steam and linearly on the area of flow.

(4) The regenerative bleed flow depends linearly on flow rate. The flow-time relationship for the turbines is represented by a first order lag.

(5) In order to take into account the reduction of rotational losses, root and tip clearance losses for the H.P. and L.P. turbines at reduced flow rates, η^*_{Hp} and η^*_{Lp}, are multiplied by correction factors η_{CFHP} and η_{CFLP}, respectively, which are assumed to depend linearly on the flow rate.

(6) The bleed flow is tapped right at the end of the high pressure turbine, and thus the entire flow through the turbine participates in producing torque while only one-half of the bleed flow is assumed to produce torque in the L.P. turbine. The second assumption is due to the fact that bleed flow is removed at various points in the I.P. and L.P. turbines.

(7) Heat exchange in the reheater is assumed to be perfect and the dynamics of mass balance and energy balance are lumped at a single point. Pressure drop in the reheater is assumed to be zero.

With these assumptions the system equations representing the turbine dynamics are given below. The variable names are explained in Nomenclature B.

(i) Nozzle Chest

$$\frac{dh_c}{dt} = \left(\frac{w_1 \bar{h}_s - w_2 h_c}{\rho_c V_c} + \frac{J\, p_c}{\rho_c^2} \frac{d\rho_c}{dt} \right) \frac{1}{1 - k_1 J} \tag{3.1}$$

$$\frac{d\rho_c}{dt} = \frac{w_1 - w_2}{v_c} \tag{3.2}$$

$$w_1 = c_1^* A_1 p \tag{3.3}$$

$$w_2 = A_{K2} (p_c \, p_c - p_R \rho_2) \tag{3.4}$$

$$x = (h_2 - h_f)/h_{fg} \tag{3.5}$$

$$\rho_2 = \frac{1}{x v_g + (1-x) v_f} \tag{3.6}$$

$$v_f = 0.0184$$

$$v_g = 2.288 - 0.0166 (p_R - 200) \tag{3.7}$$

$$h_s = 12.57 \times 10^5 - 27 (p - 1000) \tag{3.8}$$

$$p_c = \rho_c (k_1 h_c - k_2) \tag{3.9}$$

(ii) High pressure turbine

$$T_{w2} \frac{dw_2''}{dt} = (w_2 - w_{BH}) - w_2'' \tag{3.10}$$

$$w_{BHP} = K_{BHP} w_2 \tag{3.11}$$

(iii) Moisture separator

$$w_{MS} = (w_2 - K_{BHP} w_2) - w_2' \tag{3.12}$$

$$w_2' = \frac{h_2 - h_f}{hfg} w_2'' \tag{3.13}$$

$$h_f = 355 + 0.44 (p_R - 200) \times 1.055 \times 10^5 \tag{3.14}$$

$$h_{fg} = 843 - 0.4\ (p_R - 200)\ \ X\ 1.055\ X\ 10^5 \tag{3.15}$$

(iv) Reheater

(a) Main steam

$$\frac{d\rho_R}{dt} = \frac{w_2' - w_3}{v_R} \tag{3.16}$$

$$\frac{d_{hR}}{dt} = \left[\frac{Q_R + w_2' h_g - w_3 h_R}{\rho_R v_R} + \frac{Jp_R (w_2' - w_3)}{\rho_R^2 \ \bar{}_R} \right] \frac{1}{1-K_1 J} \tag{3.17}$$

$$p_R = \rho_R (k_1 h_R - k_2) \tag{3.18}$$

$$w_3 = k_3 (p_R \rho_R)^{1/2} \tag{3.19}$$

$$h_g = \left[1198.4 + 0.04\ (p_R - 200)\right]\ X\ 1.055\ X\ 10^3 \tag{3.20}$$

(b) Reheater steam

$$\frac{d\ w_{PR}^1}{dt} = \frac{w_{PR} - w_{PR}^1}{T_{R1}} \tag{3.21}$$

$$\frac{d\ Q_R}{dt} = \frac{w_{PR} + w_{PR}^1}{2\ T_{R2}\ H_R} (T_s - T_R) - \frac{Q_R}{T_{R2}} \tag{3.22}$$

$$w_{PR} = C_2^*\ A_2\ p \tag{3.23}$$

$$T_s = 544.6 + 0.125\ (p - 1000) + 460 \tag{3.24}$$

$$T_R = p_R/\rho_R\ R \tag{3.25}$$

$$R = 201.7/0.365\ (520 + 460) \tag{3.26}$$

(v) I.P. and L.P. turbine

$$\frac{dw_3'}{dt} = (1 - K_{BLP})\, w_3 - \frac{w_3'}{T_{w3}} \tag{3.27}$$

$$w_{BLP} = K_{BLP}\, w_3 \tag{3.28}$$

(vi) Heater 1

$$\frac{dh_{fw}'}{dt} = \frac{Q_{H1}}{T_{H1}\, w_{Fw}} + \frac{h_o - h_{fw}'}{T_{H1}} \tag{3.29}$$

$$Q_{H1} = H_{Fw}\,(w_{HP}' + w_{BLP}) \tag{3.30}$$

(vii) Heater 2

$$\frac{dh_{fw}}{dt} = \frac{Q_{Hz}}{T_{H2}\; w_{Fw}} + \frac{h_{fw}' - h_{fw}}{T_{H2}} \tag{3.31}$$

$$Q_{H2} = H_{Fw}\,(w_{BLP} + w_{MS} + w_{PR}^1) \tag{3.32}$$

$$\frac{dw_{HP}'}{dt} = \frac{w_2 - w_2'}{T_{H2p}} + \frac{w_{PR}^1 - w_{HP}'}{T_{H2p}} \tag{3.33}$$

(viii) work output equations

$$T_{HP} = \frac{k_T}{\eta_{HP}}\, w_2\,(h_c - h_2') \tag{3.34}$$

$$T_{LP} = \frac{K_T}{\eta_{LP}}\, w_3\,(h_R - h_4) \tag{3.35}$$

$$h_2' = \left[1067 + 0.37\,(p_R - 200) - 0.0011\,(p_R - 200)^2 - 0.1\,(p_1 - 1000)\right] X\ 1.055\ X\ 10^3 \tag{3.36}$$

$$h_4' = 898\ X\ 1.055\ X\ 10^3$$

$$\eta_{HP} = \eta_{CFHP}\,\eta^{*}_{HP} \tag{3.37}$$

$$\eta_{LP} = \eta_{CFLP}\,\eta^{*}_{LP} \tag{3.38}$$

$$\eta^{*}_{LP} = \eta^{*}_{HP} = 0.86$$

$$\eta_{CFHP} = \frac{w_2'/w_2^{*} - w_L}{(w_2'/w_2^{*})\,(1 - w_L)} \tag{3.39}$$

$$\eta_{CFLP} = \frac{w_3'/w_3^{*} - w_L}{(w_3'/w_3)\,(1 - w_L)} \tag{3.40}$$

$$K_T = 778/(1.055 \times 10^3) \tag{3.41}$$

$$\text{TORQUE} = T_{HP} + T_{LP} \tag{3.42}$$

$$\text{POWER} = \frac{\text{TORQUE} \times \Omega}{500} \tag{3.43}$$

A closer look at the equations reveal that equations (3.1), (3.2), (3.10), (3.16), (3.17), (3.21), (3.22), (3.27), (3.29), (3.31) and (3.33) are differential equations with the rest being algebraic relationships. Thus, in the turbine model there are 11 nonlinear differential equations and 32 algebraic relationships.

They can be represented in the state variable form as,

$$\dot{\underline{X}} = f(\underline{X}, \underline{U}) \tag{3.44}$$

where,

statevector, $\underline{X}^T = (X_1, X_2 \ldots X_{11})$

$$= (h_c \; w_2'' \; \rho_c \; \rho_R \; h_R \; w_{PR}^1 \; Q_R \; w_3' \; h_{fw}' \; h_{fw} \; w_{Hp}') \qquad (3.45)$$

input vector, $\underline{U}^T = (U_1 \; U_2)$

$$= (A_1 \; A_2) \qquad (3.46)$$

3.3 Low order model of the plant system

The method developed in the present work was applied to obtain low order model for the nuclear reactor turbine system described above. Although, for this system the model equations were available, in actual practice, there is no necessity for these equations to be present in order to obtain low order model for the system. Measurement of system responses at discrete intervals of time due to known excitations to the inputs is sufficient to determine low order model for the system. In the present work, this data was generated from the model equations.

As can be seen from the equations describing the plant dynamics of the system, it is not easy to find out the exact order of a transfer function relating an output variable to a particular input variable. However, looking at the number of differential equations in the system, which is eleven, it can be assumed that no element in the transfer function matrix can be of order more than eleven.

It was decided to model each element in the transfer function matrix as of second order and go for higher orders if the responses of the model did not match with the responses of the original system. However, strategy was to go for even lower order model and find out the lowest order model possible.

In practical situations if plant system equations are not available, one can decide on the order of the transfer functions and adopt the above procedure to obtain low order model.

Having decided to model each element in the transfer function matrix (2.14) as of second order, the most general second order transfer function with m = 2, h = 2 in eqn. (2.9) was selected. Thus a typical second order transfer function will look like,

$$H_{kl}(z) = \frac{a_{kl,o} + a_{kl,1} Z^{-1} + a_{kl,2} Z^{-2}}{1 - b_{kl,1} Z^{-1} - b_{kl,2} Z^{-2}}$$

A 3.3 percent step change in the main throttle valve area (A_1) and a 10 per cent step change in by-pass valve area (A_2) were made for response studies. Changes over the steady state valves in the output variables due to both of these inputs acting together. as well as individually, were taken with the sampling interval T = 0.1 sec. and using 200 samples.

These output values were put in the Data matrix (2.20) and equation (2.21) was solved for unknown coefficients by using the algorithms given in section 2.3. The computations were carried out on DEC 1090 Computer System.

The coefficients of the pulse transfer functions were converted into corresponding continuous transfer function coefficients by the method described in section 2.4.1.

Other low order models for the system with transfer function matrix having all first order transfer functions as its elements, and with transfer function matrix carrying a combination of first and second order transfer functions were also derived. Discussion of responses of these various models is given in Chapter IV.

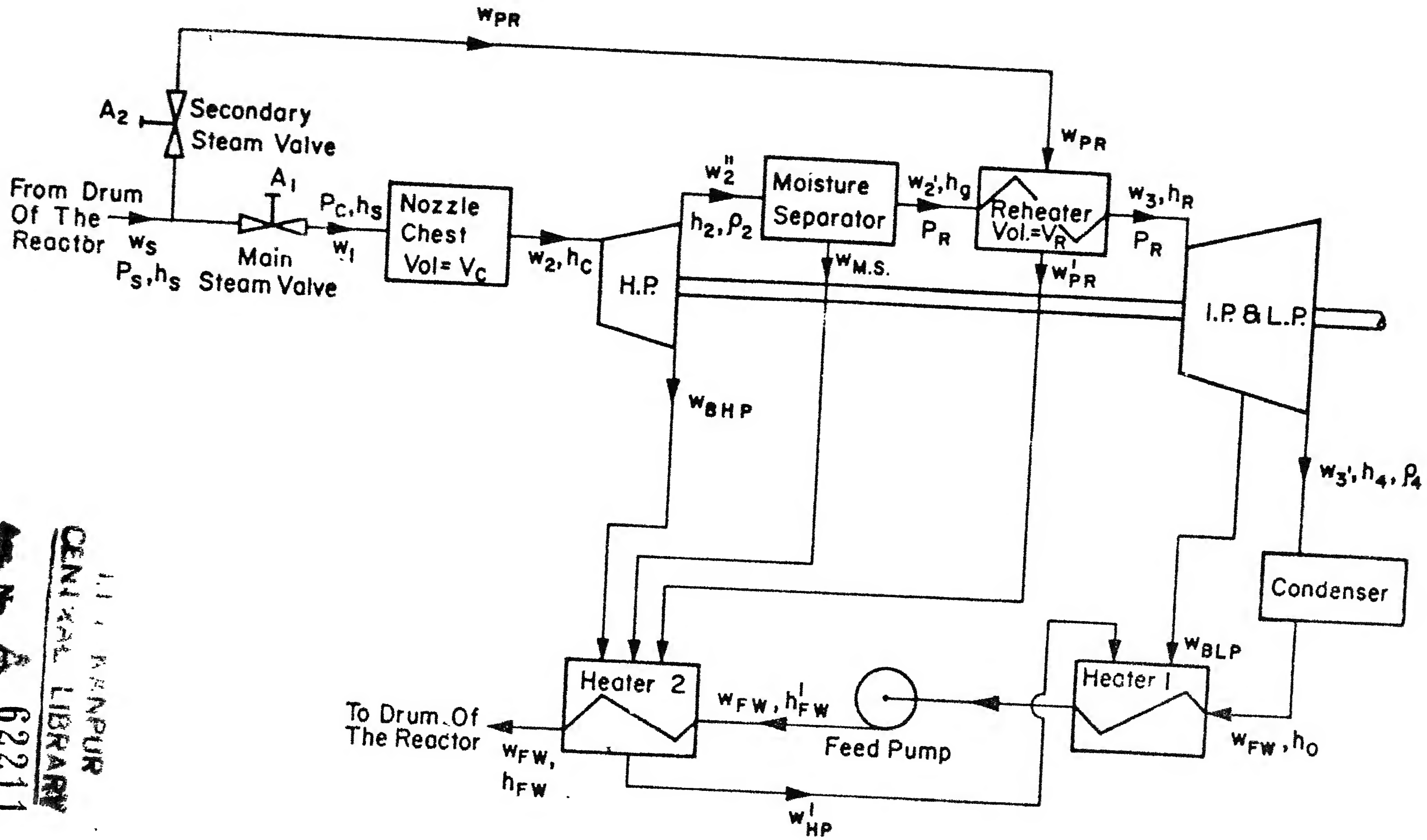

Fig. 3·1 Schematic diagram of the nuclear reacter turbine system

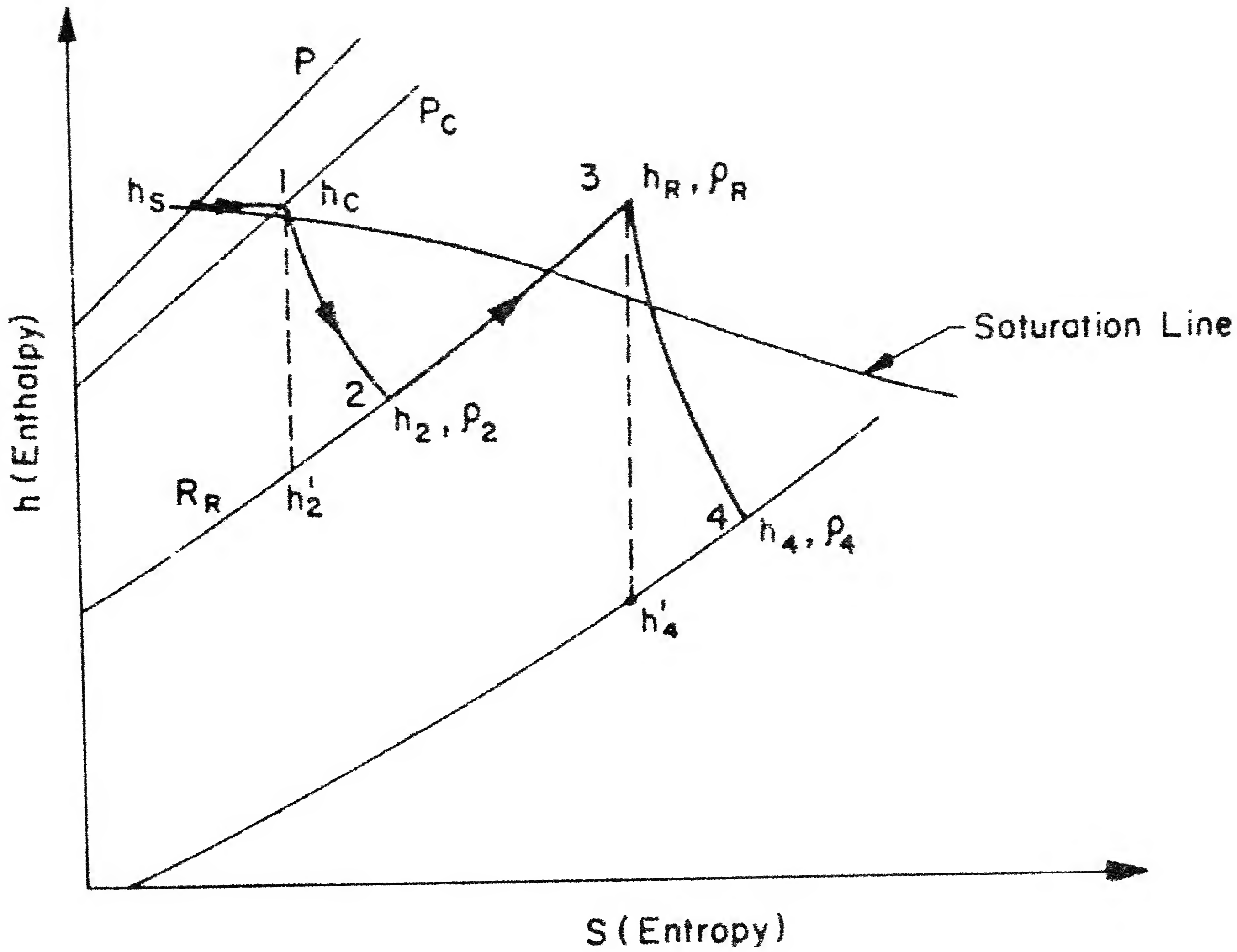

Fig. 3·2 Rankine cycle: turbine and reheater part only

CHAPTER IV

DISCUSSION ON THE RESULTS AND SELECTION OF LOW ORDER MODEL

4.1 Introduction

The present method was applied to obtain low order models of the nuclear reactor turbine system described in Chapter III. In the present chapter different low order models for the system are presented. Responses have been obtained for -

(1) Reheater temperature due to 3.3% step change in the main throttle valve area and 10% step change in by-pass valve area.

(2) Low pressure turbine torque due to 3.3% step change in the main throttle valve area and 10% step change in by-pass valve area.

(3) High pressure turbine torque due to 3.3% step change in the main throttle valve area and 10% step change in by-pass valve area.

(4) Steam flow rate due to 3.3% step change in the main throttle valve area.

3.3% and 10% step inputs are chosen only for the the sake of comparison of responses of the low order model

and the actual system, the responses of which are known for these inputs [62]. The method at such does not put any restriction on the input.

4.1.1 Reheater temperature

The four models obtained for this response are -

Model I - Both main and by-pass throttle valves having second order transfer functions.

Model II - Main throttle and by-pass valves having II and I order transfer functions respectively.

Model III- Main throttle and by-pass valves having I and II order transfer functions respectively.

Model IV - Both main throttle by-pass valves having I order transfer functions.

These models are presented in table 4.1(a) and their responses are shown in Fig. 4.1(a). In this figure, it can be seen that the best overall response is obtained by model I. However, the best peak response is obtained by model II. This is because of the non-oscillatory nature of the first order transfer function (used to model the behaviour of by-pass valve) superimposition of which tends to reduce the overall peak overshoot. The transient responses of Models III and IV are considerably in error. Magnitude of steady state

error (.07%) is same in all the models.

Table 4.3 gives time constants (I order trans. func.) and settling time (II order trans. func.). Table 4.4 gives the CPU time for model generation.

Computer Processing Unit (CPU) time for the various models are shown in Table 4.4.

4.1.2 Low pressure turbine torque

The models obtained for this response are :

Model I - Both main throttle and by-pass valves having second order transfer functions.

Model II - Transfer functions of main throttle and by-pass valves of second and first order respectively.

Model III- Transfer functions of main throttle and by-pass valves of first and second order respectively.

Model IV - Both main throttle and by-pass valves having I order transfer functions.

These models are presented in Table 4.1(b) and their responses are shown in Fig. 4.1(b). As can be seen from this figure, the best transient and steady-state (.07% error) responses are obtained by Model I. Response of Model II gives much higher steady-state error (.21%). The transient response is also not as good. Much poorer transient responses are obtained by Models III and IV. However, the latter gives much

smaller steady-state error (.25% in model IV compared to .42% in Model III). Time constants, settling time and CPU time are given in Table 4.3 and 4.4 respectively.

4.1.3 High pressure turbine torque

Four models are obtained for this response and they are :

Model I - Both main throttle and by-pass valves having second order transfer functions.

Model II - Transfer functions of main throttle and by-pass valves of second and first order respectively.

Model III- Transfer functions of main throttle and by-pass valves of first and second order respectively.

Model IV - Both main throttle and by-pass valves having I order transfer functions.

These models are given in Table 4.1 (c) and their responses are shown in Fig. 4.1(c). The figure shows that the best steady-state as well as transient responses are obtained by Model I. But the best peak response is given by Model II. This is because the peak overshoot is reduced by the non-oscillatory behaviour of the first order transfer function which is used to model by-pass valve characteristics. Model III gives higher transient and steady-state (.69%) errors, than the other two models. However, the highest steady-state

and transient response errors are given by Model IV. Time constants, settling time and CPU time are given in Table 4.3 and 4.4 respectively.

4.1.4 Steam flow rate through nozzle chest

Two models are considered for this response :

Model I - Second order transfer function

Model II - First order transfer function.

These models are given in Table 4.2 and their responses are shown in Fig. 4.2. As can be seen from the figure, Model I gives the best transient and steady-state responses. Model II gives much more transient error, however, steady-state responses of these two models do not differ much (.10% error in Model I and .15% error in Model II).

Table 4.3 gives time constants (I order trans. func.) and settling time (II order trans. func.). Table 4.4 gives the CPU time for model generation.

4.2 Selection of a low order model

In the previous section several low order models for the nuclear reactor turbine system were obtained. In this section suggestions have been made regarding the selection of a particular model.

The low order model is selected on the basis of application which it is expected to be put to. In the on-line applications like adaptive control of a system, the need is to generate in smallest possible

time a low order model, which will describe the responses of the original system with sufficient accuracy. Here, the time required to generate the low-order model is the most important factor. Thus a model with smaller number of parameters will be given preference over a model with more parameters even though its response may not be as accurate as that of the more complex model. However, in the situations where the need is just to obtain a low order model on the system, and to continuously refine it as more information on the system becomes available in the form of input-output data, the time required to generate the model is not a very important factor. In these cases, the low order model which gives the most accurate responses will be selected.

Based on these considerations we will now select low order models of the nuclear reactor turbine system.

4.2.1 Most appropriate model for on-line applications

As can be seen from Table 4.3, generation time of Model II for all the four responses - reheater temperature, low pressure turbine torque, high pressure turbine torque and steam flow rate through nozzle chest, is around 60% of that required to generate Model I. Thus for all the responses, Model II is selected and the low order model of the system for these responses

is,

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} \dfrac{8.9896\,(1 + 0.9681\,s)}{s^2 + 0.6624\,s + 0.2304} & \dfrac{8.2112}{s + .0834} \\ \dfrac{1506.1101\,(1 + 0.6605\,s)}{s^2 + 0.6624\,s + 0.2304} & \dfrac{99.8512}{s + 0.0818} \\ \dfrac{2511.8734\,(1 + 0.6831\,s)}{s^2 + 0.6562\,s + 1.2061} & \dfrac{10.034}{s + 0.0830} \\ \dfrac{5.3287}{s + 0.0840} & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

where,

C_1 - Reheater temperature over its steady state value

C_2 - Low pressure turbine torque over its steady state value

C_3 - High pressure turbine torque over its steady state value

C_4 - Steam flow rate through the nozzle chest over its steady state value

r_1 - Main throttle valve area

r_2 - By-pass valve area.

4.2.2 Most appropriate model for off-line applications

For off-line applications, time required to generate a model is not the most important factor. The model of reasonable order which best describes the system responses are selected. So, for the nuclear reactor turbine system, Model I is selected for all the responses. Hence the most suitable model for this system is,

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} \dfrac{9.0225(1+0.8614s)}{s^2+0.6952s+0.2959} & \dfrac{8.7032(1+0.1297s)}{s^2+0.6830s+0.2314} \\ \dfrac{1521.7381(1+0.4432s)}{s^2+0.6222s+0.2601} & \dfrac{100.6261(1+0.4118s)}{s^2+0.6248s+0.1936} \\ \dfrac{2499.0842(1+0.5369s)}{s^2+0.6721s+1.2544} & \dfrac{11.5254(1+0.6713s)}{s^2+0.6324s+0.8649} \\ \dfrac{5.4154(1+0.2967s)}{s^2+0.6971s+0.7225} & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

Table 4.1 (a)

Reheater Temperature

Model \ Input variable	Change in the main throttle valve area	Change in the by-pass valve area
I	$\frac{9.0225(1+0.8614s)}{s^2+0.6952s+0.2959}$	$\frac{8.7032(1+0.1297s)}{s^2+0.6830s+0.2314}$
II	$\frac{8.9896(1+0.9681s)}{s^2+0.6624s+0.2304}$	$\frac{8.2112}{s+.0834}$
III	$\frac{8.8728}{s+0.0817}$	$\frac{8.4351(1+0.1137s)}{s^2+0.5881s+0.2400}$
IV	$\frac{8.9794}{s+0.0819}$	$\frac{8.1312}{s+0.0806}$

Table 4.1 (b)

Low Pressure Turbine Torque

Model \ Input variable	Change in the main throttle valve area	Change in the by-pass valve area
I	$\frac{1521.7381(1+0.4432s)}{s^2+0.6222s+0.2601}$	$\frac{100.6261(1+0.4118s)}{s^2+0.6248s+0.1936}$
II	$\frac{1506.1104(1+0.6605s)}{s^2+0.6624s+0.2304}$	$\frac{99.8512}{s+0.0818}$
III	$\frac{2009.8716}{s+0.0809}$	$\frac{98.7312(1+0.3121s)}{s^2+0.6212s+0.2025}$
IV	$\frac{2007.0813}{s+0.0826}$	$\frac{99.2213}{s+0.0832}$

Table 4.1(c)

High Pressure Turbine Torque

Model \ Input Variable	Change in the main throttle valve area	Change in the by-pass valve area
I	$\dfrac{2499.0842(1+0.5369s)}{s^2+0.6721s+1.2544}$	$\dfrac{11.5254(1+0.6713s)}{s^2+0.6324s+0.8649}$
II	$\dfrac{2511.8734(1+0.6831s)}{s^2+0.6562s+1.2061}$	$\dfrac{10.034}{s+0.0830}$
III	$\dfrac{2331.8854}{s+0.0810}$	$\dfrac{12031(1+0.5671s)}{s^2+0.6411s+0.6400}$
IV	$\dfrac{2599.6849}{s+0.0820}$	$\dfrac{11.9813}{s+0.0813}$

Table 4.2

Steam Flow Rate Through Nozzle Chest

Model \ Input Variable	Change in the main throttle valve area
I	$\dfrac{5.4154\,(1 + 0.2967s)}{s^2 + 0.6971s + 0.7225}$
II	$\dfrac{5.3287}{s + 0.0840}$

Table 4.3

Settling time (II order trans. func.) and Time Constants (I order trans. func.)

Model	Response	R.T.	L.P.T.T.	H.P.T.T.	S.F.R.
I	II order	11.48	12.85	11.98	11.50
	II order	11.71	12.80	12.65	-
II	II order	12.08	12.17	12.10	11.90
	I order	11.99	12.22	12.20	-
III	I order	12.23	12.36	12.18	-
	II order	12.40	12.88	12.50	-
IV	I order	12.20	12.10	12.19	-
	I order	12.51	12.02	12.30	-

Abbreviations : R.T. - Reheater Temperature

L.P.T.T.- Low Pressure Turbine Torque

H.P.T.T.- High Pressure Turbine Torque

S.F.R. - Steam Flow Rate.

Table 4.4

Computer Processing Unit Time (secs.)

Model \ Input variable	R.T.	L.P.T.T.	H.P.T.T.	S.F.R.
I	92.13	91.18	92.22	60.38
II	55.25	55.51	55.61	36.43
III	55.90	55.72	55.11	-
IV	35.28	35.09	35.11	-

Abbreviations : R.T. - Reheater Temperature

L.P.T.T. - Low Pressure Turbine Torque

H.P.T.T. - High Pressure Turbine Torque

S.F.R. - Steam Flow Rate.

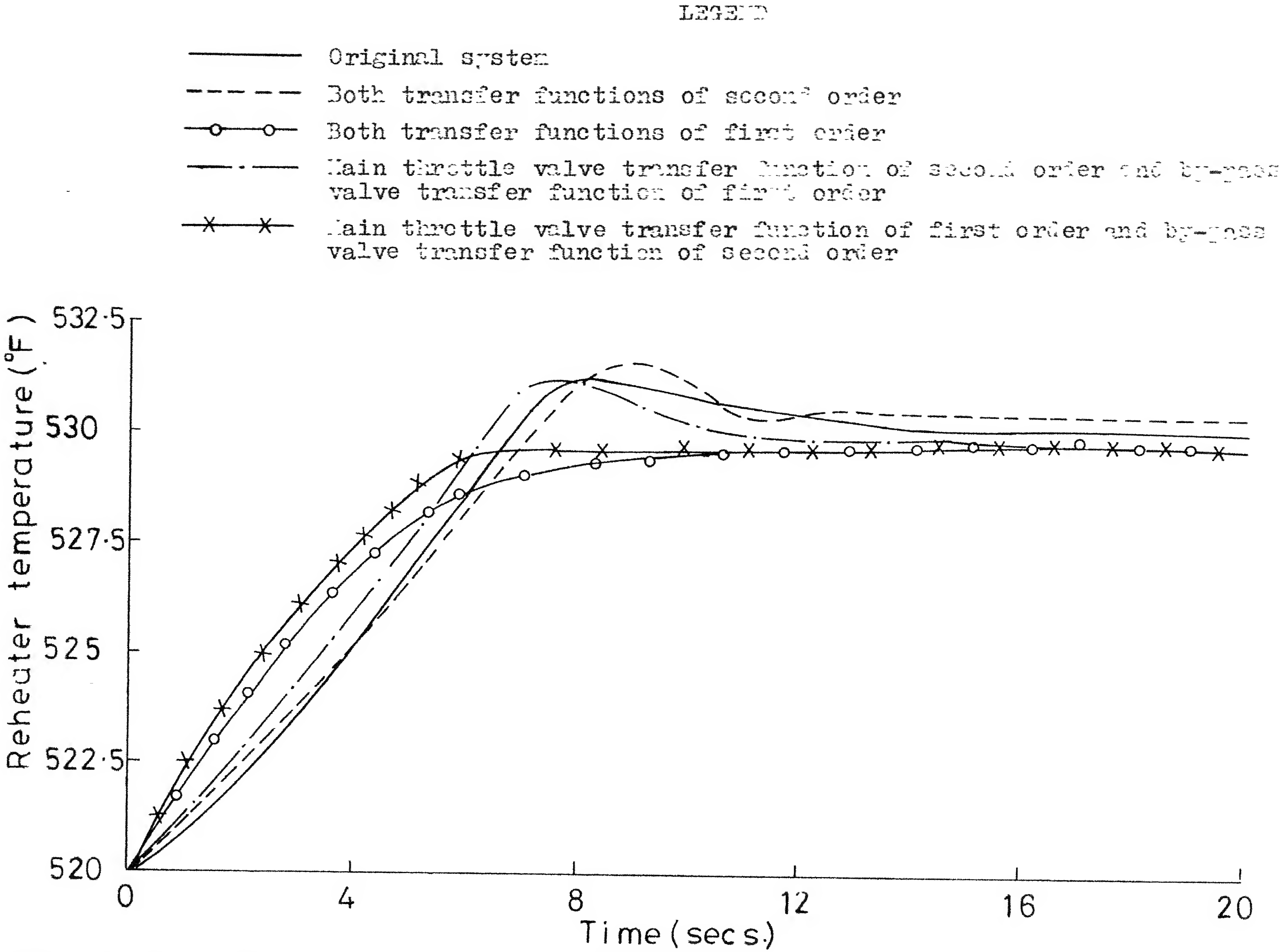

Fig. 4·1a Responses for a 3·3 % step input in main throttle valve area and a 10 % step input in by-pass valve area

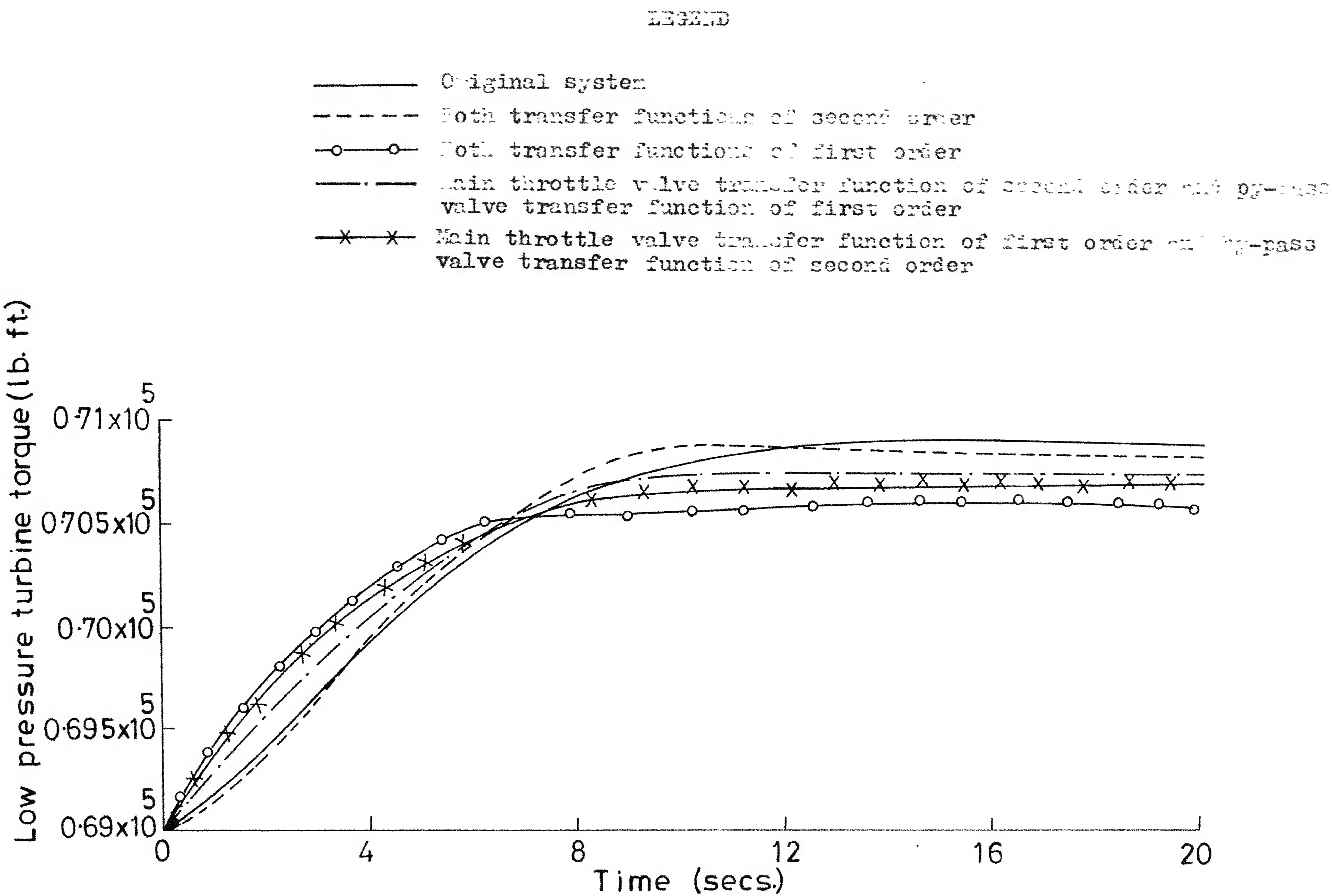

Fig. 4·1b Responses for a 3·3 % step input in main throttle valve area and a 10 % step input in by-pass valve area

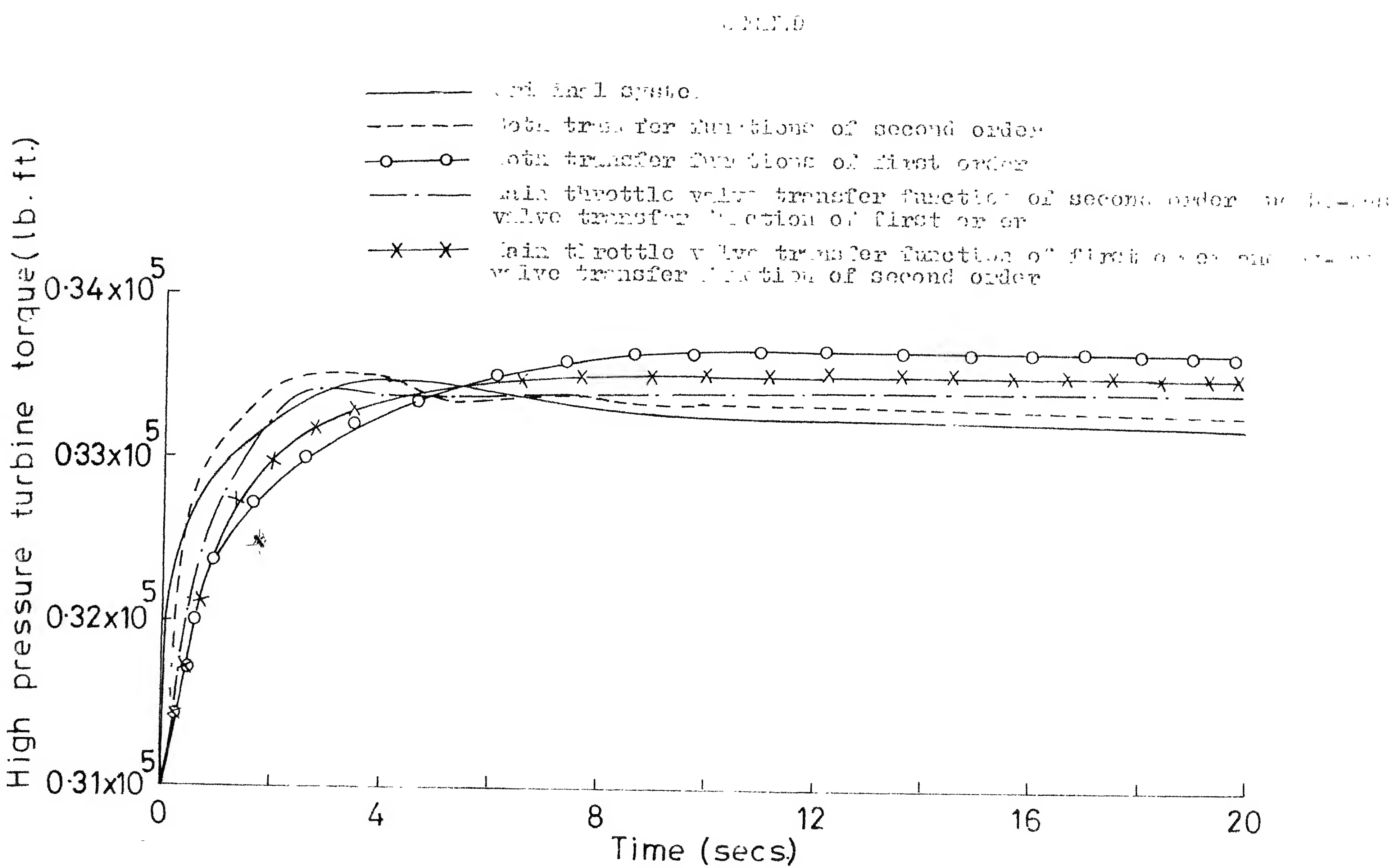

Fig. 4·1c Responses for a 3·3% step input in main throttle valve area and a 10% step input in by-pass valve area

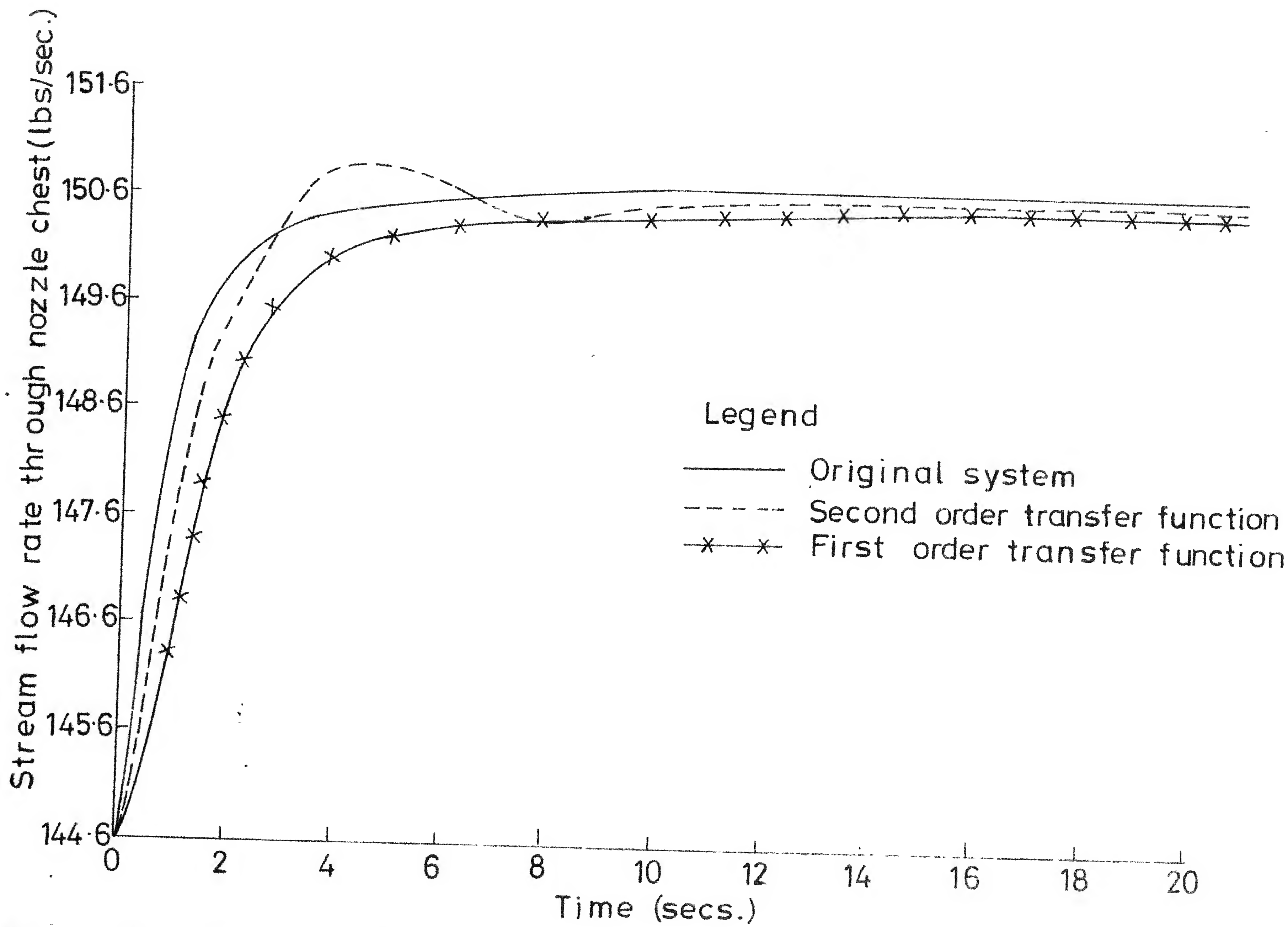

Fig.4·2 Responces for a 3·3% step input in main throttle valve area.

CHAPTER V

CONCLUSION, LIMITATIONS OF THE PRESENT METHOD AND SCOPE FOR FUTURE WORK

5.1 Conclusion

In the present work, a method has been developed to obtain low order models of large multivariable systems. The method is applicable to both linear as well as non-linear systems and does not require the transfer function or vector differential equation of the system to be specified. Only measured input-output data of the system at discrete intervals of time is required to determine its linear, low order, discrete-time model. The input-output data is put in the form of a system of difference equations, with model parameters as unknowns. The equations are solved for these unknowns by the use of matrix pseudo-inverse. The parameters thus determined, minimize the sum of the squares of the errors between the responses of the original system and its low order model at the sampling instants. A recursive algorithm is used for the solution of the system of difference equations by matrix pseudo-inverse technique. This reduces the computational and data storage requirements and enables successively refined model of the system to be determined as new data from the system becomes available.

An iterative technique developed by Smith [59] is used to obtain the corresponding continuous-time model of the discrete-time model obtained by the above method.

The method was applied to a nuclear reactor turbine system [62],[63], which has 11 non-linear differential equations. Reheater temperature, low pressure turbine torque and high pressure turbine torque responses for 3.3% step change in main throttle valve area and 10% step charge in by-pass valve area and steam flow rate through nozzle for 3.3% step charge in main throttle valve area were used to obtain the low order models for the system for these responses. The different models, with both transfer functions of II order, both transfer function of I order as well as combination of II and I order transfer functions were obtained. Responses of these models have been discussed and particular models most suitable for on-line and off-line applications have been suggested.

5.2 Limitations of the present method

The limitations of this method are as follows,

(1) Steady state error between the model and system responses cannot be predicted before obtaining the model. This is because the method minimizes the sum of squares

of the errors between the system and model responses and a fixed expression for steady state error, like the one given in Davison's method [25] cannot be derived.

(2) Least squares criterion may not give the best results in all systems.

(3) Values obtained from the iterative algorithm may not converge easily.

(4) The time constants of all the transfer functions may not be the same. Thus order of the low order model may not be what was specified.

5.3 Scope for future work

Several suggestions are given below which may make the present method more useful :

(1) In the present work, the method was applied to a nuclear reactor turbine system. It should be used to obtain low order models of several other systems to give more confidence on its applicability.

(2) The method may be extended to be able to deal with systems whose parameters vary slowly with time.

(3) A 'package' computer program can be developed which should directly give low order, continuous time model by doing the low order modelling and converting the discrete-time model into the corresponding continuous-time model. This program should also select the most appropriate low order model for a given application by considering the response errors and model generation time.

(4) The method may be modified for 'control law' reduction applications.

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APPENDIX

MATRIX PSEUDO-INVERSE

1. Concept of Matrix Pseudo Inverse

For a rectangular matrix, an inverse does not exist in the ordinary sense. It is impossible to have a unique and exact solution for a set of simultaneous equations when the number of equations are not equal to the number of unknowns. Two cases might arise :

Case (a) : Number of equations exceeds the number of unknowns. Consider the equation,

$$\underset{(n\times m)}{H}\ \underset{(m\times 1)}{\underline{x}} = \underset{(n\times 1)}{\underline{y}} \qquad (n > m) \tag{A-1}$$

where,

H is a (nxm) matrix

$\underline{x}$ is a (mx1) vector

and $\underline{y}$ is a (nx1) vector.

The inverse of H does not exist. For example, if,

$$H = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \underline{y} = \begin{bmatrix} 10 \\ 4 \end{bmatrix} \tag{A-2}$$

then eqn.(A-2) implies either

$x = 10$, or $x = 2$,

and we cannot find a solution for $\underline{x}$.

The problem, then, is to find a (mx1) vector $\underline{x}$, which approximately satisfies eqn.(A-1). The solution is given by,

$$\underline{x}' = H^L \underline{y} \qquad \text{(A-3)}$$

H^L is a non-unique (mxn) matrix called Left Inverse of H such that.

$$H^L H = I_m \qquad \text{(A-4)}$$

where,

I_m is an (mxm) identity matrix.

Letting $H^L = \left[h_1 \quad h_2 \right]$ for the system of eqns. (2) we find that,

$$h_1 + 2h_2 = 1 \qquad \text{(A-5)}$$

Therefore H^L is given by all points on the line H-H in Fig. A-1; hence H^L is not unique. The vector H is shown lying along the line P. In other words, an infinite number of $\underline{x}$'s exist, none of which are solutions of the original problem,

$$\underline{x}' = H^L \underline{y}$$

However, there exists a unique solution for the minimum Left interse, H^{LM}, given as,

$$\hat{\underline{x}} = H^{LM} \underline{y} \qquad \text{(A-6)}$$

H^{LM} exists when H has rank m or greater, and is given by,

$$H^{LM} = (H^T H)^{-1} H^T \qquad \text{(A-7)}$$

We will show that the distance between $H\underline{x}$ and $\underline{y}$ is minimized for the "best" solution $\underline{x}$:

$$\|Hx - y\| \geq \| H\hat{x} - y\| \qquad (A\text{-}8)$$

As an example, take systems of eqn. (A-2),

here, $H^T H = (1)^2 + (2)^2 = 5$

$$H^{LM} = (0.2)\begin{bmatrix}1 & 2\end{bmatrix} = \begin{bmatrix}0.2 & 0.4\end{bmatrix}$$

$$\underline{\hat{x}} = \begin{bmatrix}0.2 & 0.4\end{bmatrix}\begin{bmatrix}10 \\ 4\end{bmatrix} = 3.6$$

$H\underline{\hat{x}}$ for this case is given by point p_o in Fig. A-1. P_o among all points on pp, is closest to the tip of $\underline{y}$. H^{LM} lies at the intersection of the lines HH and PP.

To prove inequality (A-8), we let $\underline{x}$ be any solution and $\underline{\hat{x}}$ be the minimal norm solution, and start with the equality,

$$\begin{aligned}\|H\underline{x} - \underline{y}\| &= \| H(\underline{x} - \underline{\hat{x}}) + H\hat{x} - y\| \\ &= \|H(\underline{x} - \underline{\hat{x}}) + (HH^{LM} - I_n)\,\underline{y}\| \\ &= \|H(\underline{x} - \underline{\hat{x}})\| + \|(HH^{LM} - I_n)\,y\| + \\ &\quad 2\left[H(\underline{x} - \underline{\hat{x}})\right]^T (HH^{LM} - I_n)y \qquad (A\text{-}9)\end{aligned}$$

The last term of right-hand side of eqn. (A-9) vanishes:

$$[H(\underline{x} - \hat{\underline{x}})]^T (HH^{LM} - I_n)\,\underline{y} = (\underline{x} - \hat{\underline{x}})^T H^T (HH^{LM} - I_n)\underline{y}$$

$$= (\underline{x} - \hat{\underline{x}})^T (H^T HH^{LM} - H^T)\underline{y}$$

$$= (\underline{x} - \hat{\underline{x}})^T [H^T - H^T]\,\underline{y}$$

$$= 0.$$

Therefore, eqn. (A-9) reduces to,

$$\|H\underline{x} - \underline{y}\| = \|H(\underline{x} - \hat{\underline{x}})\| + \|HH^{LM}\underline{y} - \underline{y}\|$$

$$= \|H(\underline{x} - \hat{\underline{x}})\| + \|H\hat{\underline{x}} - \underline{y}\| \qquad \text{(A-10)}$$

Since the first term on the right-hand side of this equation is non-negative, the inequality (A-8) holds.

Case (b) : The number of unknowns, exceeds the number of knowns. Consider the system of equations,

$$\underset{(m\times n)}{H}\ \underset{(n\times 1)}{\underline{x}} = \underset{(m\times 1)}{\underline{y}} \qquad (n > m) \qquad \text{(A-11)}$$

We cannot solve for $\underline{x}$, because, the determinant $|H| = 0$. The problem, then, is to find a (nx1) vector $\underline{x}$ such that eqn. (A-11) is satisfied.

The solution to this problem is given by,

$$\underline{x} = H^R\underline{y} \qquad \text{(A-12)}$$

The (nxm) matrix H^R in this expression is called the Right Inverse of H. H^R exists when the rank of H is m or greater and is determined by,

$$HH^R = \underline{I}_m$$

where, $\underline{I}_m$ is the (mxm) identity matrix.

As in the case (a), H^R and hence the solution (A-12) is not unique. There exists, however, a unique solution, for which the norm of $\underline{x}$ is minimal. Such a solution $\hat{\underline{x}}$, called the Minimal Solution, is determined by the Minimum Right Inverse, H^{RM} so that,

$$\hat{\underline{x}} = H^{RM}\,\underline{y} \tag{A-13}$$

where,

$$H^{RM} = H^T\,(\,H\,H^T)^{-1} \tag{A-14}$$

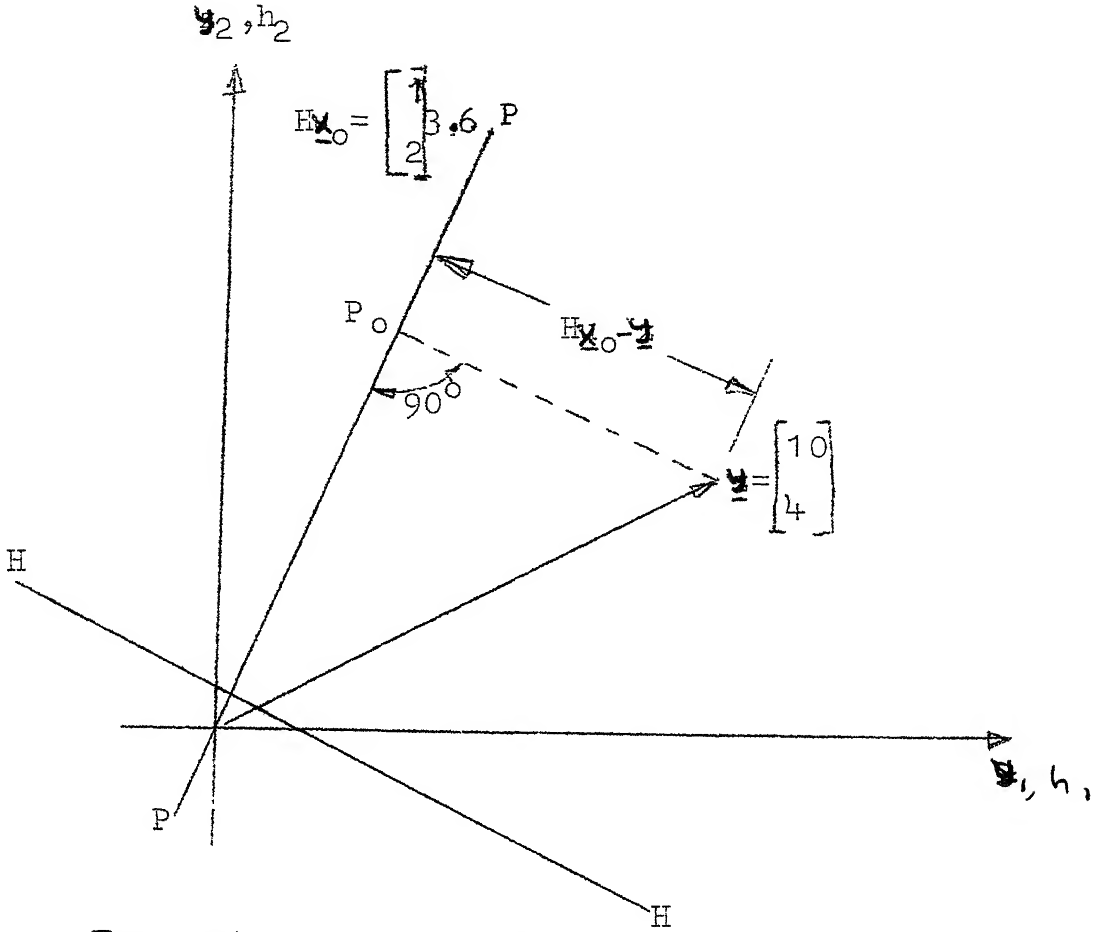

Fig. A-1 Examples of the minimum left inverse